COMPUTERIZED SCHEDULE CONSTRUCTION FOR AN AIRLINE TRANSPORTATION SYSTEM

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I. INTRODUCTION

As part of a continuing study of the application of V/STOL aircraft to the transportation problems of the Northeast Corridor, the U.S. Department of Commerce has requested that detailed data be developed on schedules, travel times, and fares which might be expected for a V/STOL system operating in the year 1980. This section deals with the computer methods used to construct such schedules.

A schedule (or more properly a schedule plan) is a complete description of a transportation system. It details the services to be offered in the dimensions of time and geography, gives the routings followed by vehicles, and indicates the loadings to be placed upon terminals. A complete statistical summary of the operation of the transportation system can be obtained once the schedule is completed. The number of vehicles and crews, their daily utilization, the expected load factors, the required number of loading gates, the average length of vehicle hop, etc. are all implicitly determined by the schedule plan. Constructing and maintaining an efficient schedule is the main problem of transportation system managements. It is both their production plan and their product to be marketed, and the economic success of the plan is gauged by the management's ability to produce a low cost production which will be saleable to the travelling public.

The use of computers in scheduling is not widespread at this time, and if they are used, it is generally for data processing as distinct from decision making or problem solving. The reasons for this are clear. There has not been in the past, sufficient capability either in the hardware, or the software to handle problems of the size and complexity associated with even such relatively small transportation systems as the airline systems. This situation has been changing in the last few years, to the point where we can now begin to handle fairly large scale scheduling problems, introducing optimization at several points, and constructing fairly quickly and easily full system schedules and their statistical summaries. Parametric investigations of the effects of restricting fleet size, terminal size, etc. can be quickly carried out. Various strategies or policy decisions are similarly easily investigated.

The construction of computer programs for the scheduling process immediately points out the need for detailed accurate data concerning demand. This is now becoming available to the airlines through their reservation systems and the management information systems evolving from them. We need to know, for example, detailed information on the number of people travelling from A to B throughout the day, by day of week and month of year, with an accuracy much greater than our present estimates. We would like to know the demand elasticity, e.g. the change in the number of people travelling as services are changed in time or quality for every service pair in the system. The character of the available data (as distinct from opinion) determines the type of problem that operations research and the computer will be able to successfully solve. Various large scale econometric models are conceivable, if revenue and cost data are available.

This section will describe the work which has been carried out for a hypothetical Airbus short haul V/STOL system in the Northeast Corridor. It is only a beginning as valuable extensions are yet to come as more applications come into the open literature. The main presentation will describe the computerized processes developed to construct a schedule plan assuming certain demand and operating data. An interesting extension showing the application of network flow theory to a more detailed representation of the schedule is then described in Appendix A.

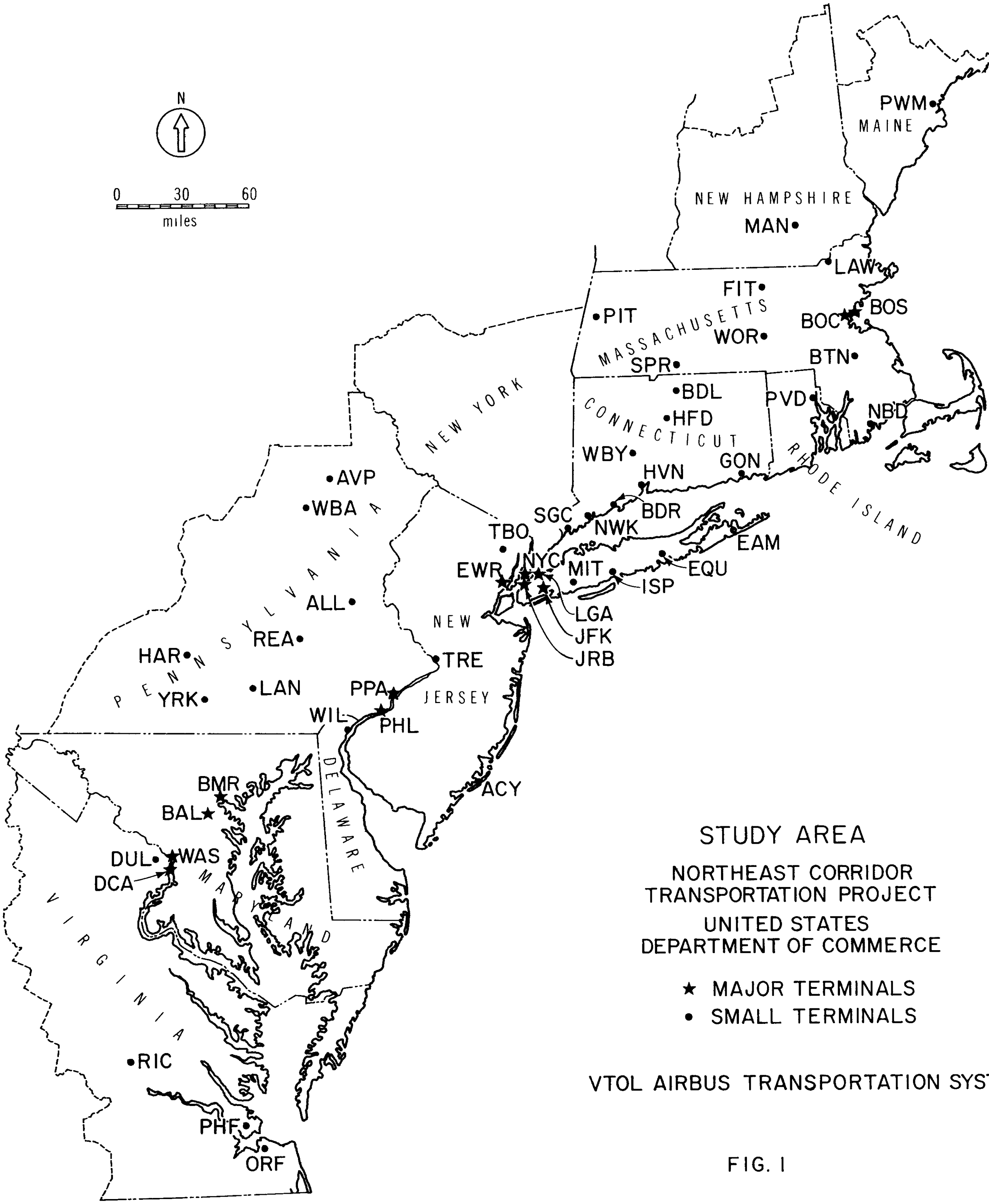
A map of the corridor showing the terminals selected for the Airbus system is shown in Figure 1. Table I shows some typical distances, travel times, and projected fares for the 1980 Airbus.

TABLE I

Typical Fares and Travel Times

VTOL Airbus System

TRIP	Distance	Time	Fare
From Washington downtown	(st. miles)	(minutes)	(1966 dollars)
to - Dulles	26	6	3.10
- Baltimore downtown	33	7	3.35
- Philadelphia downtown	123	22	6.55
- New York downtown	203	34	9.35
- Norfolk	144	25	7.25
- Providence	351	58	14.60
From Boston downtown			
to - Worcester	40	8	3.60
- Providence	46	9	3.80
- Hartford	89	16	5.35
- Laguardia	181	31	8.60
- Philadelphia downtown	266	44	11.60
- New York downtown	190	32	8.90



STUDY AREA

NORTHEAST CORRIDOR
TRANSPORTATION PROJECT

UNITED STATES
DEPARTMENT OF COMMERCE

★ MAJOR TERMINALS
• SMALL TERMINALS

VTOL AIRBUS TRANSPORTATION SYSTEM

FIG. I

II. DEMAND DATA GENERATION

The methods of generating demand data have previously been described in Reference 3. A brief description of these methods will be given here along with some detailed description of the data for a second demand assumption.

The computerized methods of timetable construction of this report assume the availability of explicit , detailed information about the passenger demand for services between any two system points. Accordingly, a method of generating such data was developed using a modified "gravity" model. The model uses projected 1980 populations for each system point (see Table 2), and the distance between points to calculate the number of passengers per day between each pair of points.

The model has the form:

$$d_{ij} = K \cdot \left[\frac{P_i \; P_j}{S_{ij}^{\alpha}} \right] \times \left[1 - e^{-(C \; S_{ij})^{\beta}} \right]$$

where K is a scaling constant; and the first bracket is the gravity model with $\alpha = 0.4$. The second bracket represents a modification to represent the share of air travel in competition with the automobile, bus, and train, etc. The constant

TABLE 2. NORTHEAST CORRIDOR AIRBUS TERMINALS, POPULATIONS, AND OPERATIONS PER DAY

Terminal Locations	Code Desig-nation	1980 Population	Passenger Orignations Per Day	Aircraft Operations Per Day
MAJOR TERMINALS				
Boston, Mass.-Logan	BOS	1,478,500	3770	168
Boston, Mass.-in or near downtown	BOC	1,508,800	2830	154
New York, NY - John F. Kennedy Internat'l	JFK	2,818,800	4252	240
New York, NY - La-Guardia Airport	LGA	3,801,600	5641	798
New York,NY - Wall Street Heliport	JRB	1,097,000	1632	126
New York,NY - Pan American Bldg.	NYC	1,701,700	2532	272
Newark, NJ - Newark Airport	EWR	1,933,800	2942	204
Philadelphia, Pa. - Philadelphia Airport	PHL	2,413,900	4537	298
Philadelphia, Pa. - downtown on the river	PPA	3,653,100	6760	646
Baltimore, MD - Friend-ship Airport	BAL	757,700	1458	601
Baltimore, Md - in or near downtown	BMR	1,850,900	3591	300
Washington, DC - Wash. Natnl Airport	DCA	1,162,00	2211	112
Washington, DC - downtown	WAS	1,801,956	3430	244
OTHER TERMINALS				
Portland, Me. - Portland Airport	PWM	136,000	249	12
Manchester, NH - Gernier Airport	MAN	111,700	218	10
Lawrence-Haverill,Mass. Lawrence Airport	LAW	198,400	377	42
Fitchburg,Mass. - Fitchburg Airport	FIT	100,000	197	28
Pittsfield, Mass. - Pittsfield Airport	PIT	90,800	.203	10
Worcester, Mass. - Worcester Airport	WOR	368,100	734	76
Brockton, Mass.- Brockton Airport	BTN	232,900	445	18
Providence, R.I. - Providence Airport	PVD	941,500	1891	104
New Bedford, Mass.- New Bedford Airport	NBD	146,100	193	46

TABLE 2. CONTINUED

Terminal Locations	Code Designation	1980 Population	Passenger Originations Per Day	Aircraft Operations Per Day
Reading, Pa. - Reading Airport	REA	319,500	667	132
Harrisburg, Pa. - Harrisburg Airport	HAR	431,300	1037	48
Lancaster, Pa. - Lancaster Airport	LAN	391,900	814	108
York, Pa. - York Airport	YRK	329,900	694	56
Trenton, NJ - Trenton Airport	TRE	357,900	635	30
Atlantic City, NJ - Atlantic City Airport	ACY	239,600	509	24
Wilmington, Del. - Wilmington Airport	WIL	631,200	1337	58
Washington, DC - Dulles Airport	DUL	776,900	1501	64
Richmond, Va. - R. E. Byrd Flying Field	RIC	633,000	1246	76
Newport News - Hampton VA - Civil Airport	PHF	570,500	1107	96
Norfolk, Va. - Norfolk Airport	ORF	972,500	1868	82
Springfield, Mass.- Springfield Airport	SPR	636,100	1336	84
Hartford, Conn. - Rentschler Airport	HFD	693,700	1431	168
Hartford-Springfield - Bradley Field	BDL	182,000	380	36
Waterbury, Conn. - Waterbury Airport	WBY	250,300	506	22
New London, Conn - New London Airport	GON	254,700	535	84
New Haven, Conn. - New Haven Airport	HVN	416,600	804	140
Bridgeport, Conn. - Bridgeport Airport	BDR	499,600	908	70
Norwalk, Conn. - SW of downtown near water	NWK	196,800	331	42
Stamford-Greenwich, Conn.-Between Stamford & Greenwich near wat.	SGC	796,200	1301	78
New York, NY - Teterboro Airport	TBO	1,023,200	1555	80
Long Island, NY - Mitchell AFB (abandoned)	MIT	1,627,200	2508	216
Islip, Long Island, NY - MacArthur Field	ISP	417,700	741	54
East Quogue, Long Is. NY- Suffolk Cty AFB	EQU	292,200	628	44
East Hampton, Long Is. NY - Airport	EHM	126,100	266	22
Scranton, Pa.-Scranton Airport	AVP	194,200	403	32
Wilkes-Barre, Pa. - Wilkes-Barre Airport	WBA	271,900	634	42
Allentown, Pa. - Allentown Airport	ALL	621,700	1259	76

C is chosen such as to cause a specific peaking of the d_{ij} variation with distance (e.g. for 200 miles C = .007, for 100 miles, C = .015). The value of β is taken as 2.0.

This data is not a prediction - it is a hypothetical pattern of demand generated to examine in detail the problems of producing a schedule or timetable by computer, and to allow some idea of desirable vehicle sizes, terminal sizes, number of operations, vehicle and terminal utilizations, etc. to be obtained. Both the scale and the pattern of demand can be changed in order to examine the sensitivity to such changes of certain operational information arising from the system schedule. It is expected that detailed projections of Northeast Corridor demands from other Department of Commerce studies will become available at some future date. As a result of the studies reported in Reference 3, a second pattern of demand has been generated with the peak of the pattern shifted from 200 to 100 miles. See Figure 2. Such a shift assumes that the air system will have a larger share of the total travel market, particularly for trips below 200 miles. Since the previous results indicated appreciable time savings, and lower costs, this shift in demand was indicated in generating a second demand pattern.

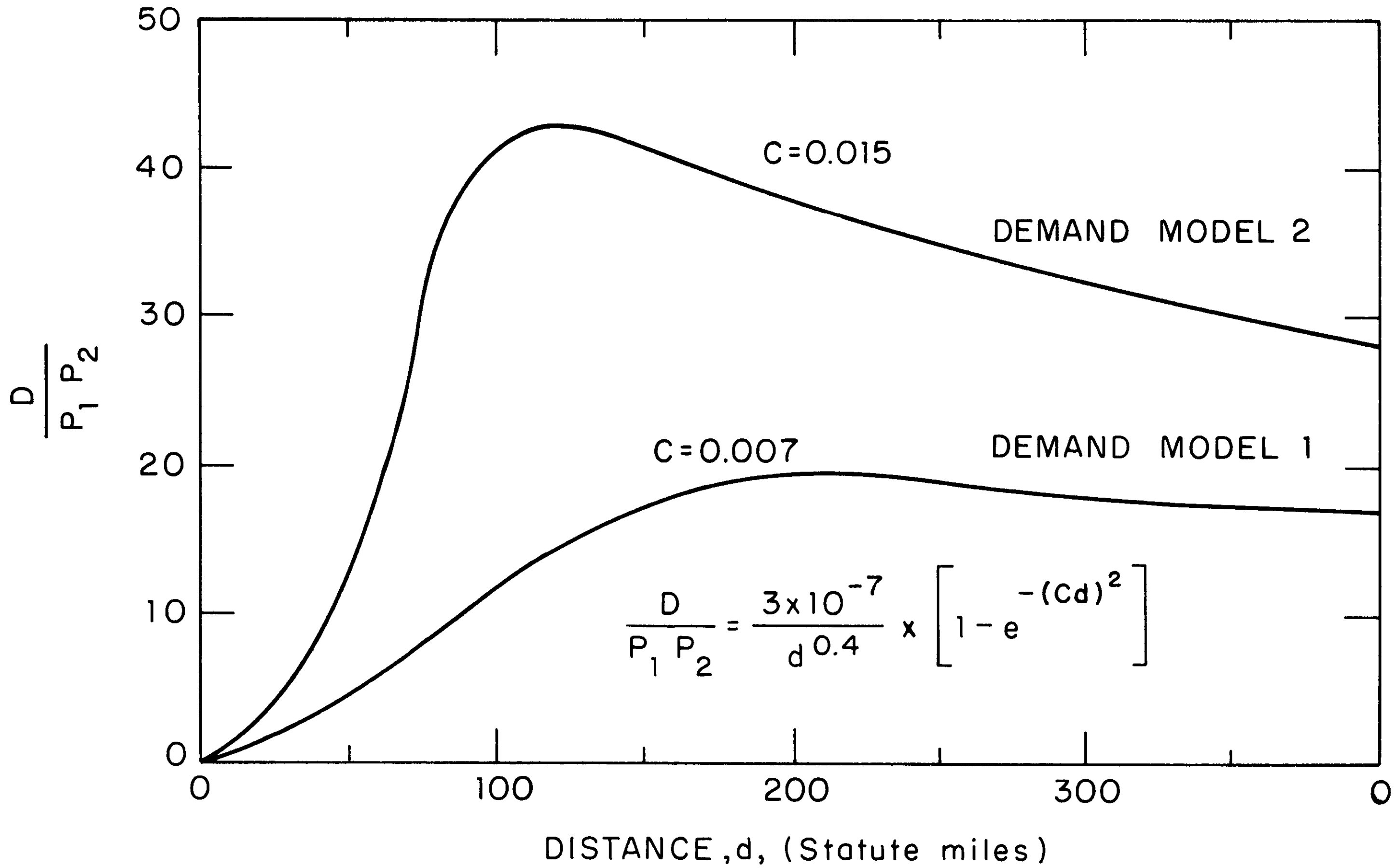


FIGURE 2 ASSUMED N.E. CORRIDOR DEMAND FUNCTION FOR AIRBUS TRAVEL

The demand data is given in Table 3. It is a 50 x 50 matrix called the D_2 matrix, with a subscript 2 to indicate the second demand pattern. The data is O & D data (origination and destination) where the matrix entry d_{ij} gives the number of trips per day from point i to point j. As is generally true for passenger traffic (but not cargo or freight), the matrix of Table 3 is symmetric, where $d_{ij} = d_{ji}$. The diagonal consists of zeroes, and the sum of any row represents the number of trips out of a given point, i. Similarly the sum of any column represents the number of trips ending at a point j. Table 2 gives the number of passenger originations for each station for the D_2 matrix. The second demand generates 27.7×10^6 passengers per year compared to 16.7×10^6 for the first demand matrix, D_1. Each entry, d_{ij}, represents a best estimate of the mean or average of daily travel demands on the system over some period such as a peak month or season. Daily demand varies from day to day in recognized seasonal and weekly cyclic patterns, and is a probabilistic or stochastic variable. In this study, d_{ij} represents the mean of such variations over all days of a peak month or season, and a system load factor is assumed later to be 60% to ensure that daily demands above the average

TABLE 3 - DAILY TRIPS BETWEEN CITY PAIRS - SECOND DEMAND

	DCA	DUL	BAL	BOS	BOC	MAN	WOR	NBD	PVD	WIL	PHL	TRE	REA	LAN	ALL	HAR	WBA	SPR	PIT	BDL	JFK	LGA	JRB	EWR	BDR	HVN	WBY	GON	HFD	ACY	BMR	WAS	MIT	TBO	NYC	ORF	RIC	PPA	PHF	YRK	AVP	SGC	NWK	ISP	EQU	EHM	PWM	FIT	BTN	LAW
DCA		19	28	136	139	10	35	14	91	88	328	47	43	50	83	61	34	64	9	18	333	449	131	235	54	44	26	26	71	32	95	4	189	122	203	131	81	497	78	38	24	90	22	46	30	13	12	9	22	18
DUL			32	90	92	7	23	9	60	61	219	31	29	34	55	41	23	42	6	12	217	293	86	153	36	29	17	17	47	21	94	37	123	79	132	86	55	329	52	25	16	59	14	30	20	9	8	6	14	12
BAL				92	94	7	24	9	61	44	198	32	26	26	55	34	23	43	6	13	230	309	91	162	37	30	18	17	48	21	7	31	130	84	140	83	56	310	50	18	17	62	15	32	21	9	8	6	15	12
BOS					4	9	30	13	86	89	326	51	42	50	87	60	38	97	16	29	446	603	172	298	86	72	43	40	112	33	226	212	263	161	268	107	69	500	63	41	27	133	33	70	51	22	22	7	7	7
BOC						9	27	13	81	91	335	52	44	52	89	62	39	95	16	29	459	621	176	307	88	73	44	40	111	33	231	217	270	166	275	109	71	513	64	42	28	136	34	72	51	22	23	7	6	7
MAN							3	2	11	7	24	4	3	4	7	5	3	7	1	2	33	45	13	22	6	5	3	3	9	2	17	16	19	12	20	8	5	37	5	3	2	10	2	5	4	2	1	0	2	0
WOR								4	18	23	86	14	11	13	23	16	10	11	3	4	119	161	46	80	21	16	9	8	17	9	59	55	70	43	72	27	18	132	16	11	7	34	8	18	12	5	6	1	5	5
NBD									3	9	33	5	4	5	9	6	4	10	2	3	46	62	18	31	9	7	4	3	11	3	23	22	27	17	27	11	7	51	6	4	3	13	3	7	4	2	2	1	1	3
PVD										60	222	35	29	34	59	40	25	49	10	14	307	414	118	206	52	40	24	12	51	22	151	141	179	111	184	71	46	341	42	27	18	88	22	45	25	10	15	7	7	17
WIL											38	18	14	16	37	32	22	43	6	13	224	303	87	150	38	31	18	17	48	12	95	133	130	81	135	70	47	87	42	21	15	63	15	32	21	9	7	6	14	12
PHL												32	51	71	104	124	75	159	23	46	744	1009	278	464	140	114	67	65	179	33	457	507	447	267	438	239	160	36	143	85	53	225	56	118	77	33	27	23	52	43
TRE													10	14	13	20	11	25	4	7	79	107	27	41	21	17	10	10	28	6	77	74	53	28	44	34	22	30	20	14	8	30	8	17	12	5	4	4	8	7
REA														3	7	9	7	21	3	6	105	141	40	69	18	15	9	8	23	9	61	67	61	37	62	30	21	79	18	8	5	30	7	15	10	4	4	3	7	6
LAN															18	6	11	24	4	7	129	174	50	89	21	17	10	10	27	11	56	76	73	47	78	38	26	117	23	5	8	35	9	18	11	5	4	4	8	7
ALL																30	11	43	6	13	185	242	65	103	36	30	18	17	48	16	134	129	112	60	102	57	38	145	34	22	8	56	14	30	20	9	7	6	14	12
HAR																	14	29	4	8	152	206	60	107	25	20	12	11	32	13	78	94	86	56	93	46	32	192	28	2	10	41	10	21	13	6	5	4	10	8
WBA																		19	3	5	90	119	34	58	16	13	8	7	21	8	58	54	52	31	53	23	16	112	14	10	0	25	6	13	8	4	3	3	6	5
SPR																			3	1	209	280	82	144	28	20	7	12	9	16	107	99	119	76	126	49	32	224	29	19	14	54	13	29	20	8	10	4	15	13
PIT																				1	30	40	12	21	5	4	2	3	5	2	15	14	17	11	18	7	5	35	4	3	2	8	2	4	3	1	1	1	2	2
BDL																					59	78	23	41	7	4	1	3	1	5	31	29	33	21	36	14	9	71	8	6	4	14	3	8	5	2	3	1	4	4
JFK																						66	30	108	88	96	69	81	218	73	570	522	28	52	53	247	161	1072	146	103	64	71	21	50	86	36	35	31	72	59
LGA																							17	91	111	127	89	110	290	102	769	702	68	28	21	331	216	1457	195	139	84	75	25	76	120	50	48	42	97	79
JRB																								11	39	41	28	32	86	29	225	206	32	6	2	96	63	394	57	41	24	31	10	27	36	15	14	12	28	23
EWR																									79	78	51	58	155	49	403	368	99	14	20	171	112	645	101	73	41	71	22	58	65	28	24	21	48	39
BDR																										2	4	9	21	14	92	85	37	34	57	41	27	213	24	17	11	8	1	5	11	4	7	6	14	11
HVN																											3	6	12	11	75	69	46	37	62	34	22	175	20	14	9	15	3	7	7	2	6	5	11	10
WBY																												5	4	7	45	41	36	24	41	20	13	103	12	8	6	12	3	8	7	3	4	3	7	6
GON																													9	7	43	40	45	30	50	20	13	100	12	8	5	20	5	9	3	1	4	3	6	6
HFD																														18	119	111	119	79	132	54	35	275	32	22	15	49	11	26	17	6	10	6	17	16
ACY																															51	50	44	28	45	24	16	50	14	9	5	22	6	12	8	3	3	2	5	4
BMR																																123	323	208	348	201	137	730	121	41	41	154	37	79	51	22	19	16	36	30
WAS																																	295	191	318	203	128	771	120	56	38	141	34	73	47	21	18	15	34	28
MIT																																		41	50	141	92	657	83	58	37	26	8	18	46	19	21	18	42	34
TBO																																			6	89	58	382	53	38	22	24	9	27	34	14	13	11	26	21
NYC																																				148	97	624	88	63	37	43	14	42	56	23	21	19	43	35
ORF																																					58	357	12	32	17	67	16	35	24	10	9	7	17	14
RIC																																						238	24	23	11	44	11	23	15	7	6	5	11	9
PPA																																							213	133	80	337	84	179	119	51	41	35	80	66
PHF																																								20	10	40	10	21	14	6	5	4	10	8
YRK																																									7	28	7	14	9	4	3	3	6	5
AVP																																										18	4	9	6	3	2	2	4	4
SGC																																											1	10	22	9	10	9	21	17
NWK																																												2	5	2	3	2	5	4
ISP																																													8	3	5	5	11	9
EQU																																														0	4	3	8	7
EHM																																															2	1	3	3
PWM																																																2	4	3
FIT																																																	2	1
BTN																																																		3
LAW																																																		

are accommodated. Variations of demand with time of day are accounted for by choosing departure times in the timetable construction.

The demand data assumed available here is similar to that obtained by the CAB in its present method of sampling ticket sales and recording the O & D flow by all airlines between various points. If the airbus system were operating with a computerized reservation system or more precisely a management information system, such data could be continuously gathered and made available for any time period. Future projections could then be based on this data in planning schedules for future time periods.

Often, a more detailed description of demand can be available for purposes of schedule planning. For example, the weekly cycle of demand could be studied with the goal of providing different daily timetables, or a semi-weekly schedule; or, competitive factors between companies or between modes could be available. No such complexities have been allowed here. The object has been to provide some idea of the geographic distribution of daily demands in order to determine the frequency of service pattern for the assumed system. This process will be described in the next two sections.

III. PASSENGER FLOW PATTERN

The demand matrix, D, gives the number of trips per day from City A to City B. Unless the system is providing direct non-stop service between all city pairs this will not be the totality of passengers using the route A-B since other "through" passengers will use the route in going X - A - B - Y. For 50 cities there are 2450 possible non-stop services, but not all of them will generate sufficient demand to warrant non-stop service. This can be seen by examining the D_2 matrix. On most airline systems only about 15-20% of such possibilities are economically attractive, so that there is a substantial percentage of through passengers on most direct services.

It is necessary to have some method of determining the passenger flow patterns for a given subset of non-stop services. The total passenger flow (non-stop plus through passengers) then determines the daily number of non-stop seats required, or the frequency of daily non-stop service required for a given vehicle seat size and load factor. This process is sometimes called determining the frequency of service pattern for the system. Lesser routes are dropped to zero frequency in favor of higher frequencies elsewhere.

This section will describe the techniques used to produce a passenger flow pattern (the F matrix) given the subset of routes to be serviced. We are interested in matching available seats/day against passengers/day on each route to be serviced in order to obtain the frequency pattern (the N matrix)in the next section. In other words, we are trying to determine on a system wide basis a balanced allocation of available seats to the geographic patterns of demand.

A multi-commodity network flow computation is used to re-route the passengers from X to Y via the shortest routing X - A - B - Y. Initially, an assumption has been made that least distance can be used as a criterion for determining the "shortest" routing. It is probably a good representation of least time when the frequency of service on all routes is fairly high, and the model can be extended (as shown in Appendix A) to be precisely least time when the time of day network and a discrete timetable are used. It is assumed that a passenger will use the services which have the earliest arrival at Y, and that the indirect routings will not have an effect on the D matrix demands.

Neither of these assumptions can be rigorously defended. In the airbus system, it is planned that knowledge of the earliest available arrival at his destination via indirect routings will be available to the passenger through the system's computer, and that intermediate stops will delay the flight only a few minutes. Both of these factors will tend to make the assumptions more applicable to airbus service than to present airline service.

Methodology for Determination of F Matrix

The methods used to determine shortest paths and the passenger F matrix are drawn from network flow theory, and the theory of graphs. A fuller theoretical understanding of these concepts can be obtained by reading Reference 4, and then Reference 5. A brief explanation of the techniques will be given here.

The computer program used was a modification of an IBM SHARE library coding by Dick Clasen at Rand Corporation for the "Out of Kilter" OKF algorithm of Ford and Fulkerson. This algorithm is applicable to large transportation network problems where the number of arcs and nodes can be 4500, and 1500 respectively. Solutions give integer values of x_{ij} and are obtained within 10 minutes of computation on an IBM 7094.

Suppose we have a network or graph G which consists of nodes (i,j,k...) and directed arcs (ij, jk,...) such that closed paths or circuits exist in the graph. With each arc ij, there are three associated scalar quantities;

l_{ij} = lower limit on arc flow

u_{ij} = upper limit of arc flow = capacity

c_{ij} = unit cost of flow from i to j and the variable x_{ij} which is called the flow of some commodity from node i to node j.

We wish to determine a flow $X = \left\{ x_{ij}, \ x_{jk}, \ldots \right\}$ such that:

1) $l_{ij} \leqslant x_{ij} \leqslant u_{ij}$ Capacity constraints for all arcs.

2) $\sum_{(ij,\ jk)} (x_{ij} - x_{jk}) = 0$ Conservation of flow at nodes j.

3) x_{ij} are integer values

and which will minimize the total flow cost,

$$Z = \sum_{\text{arcs } ij} c_{ij} \cdot x_{ij}$$

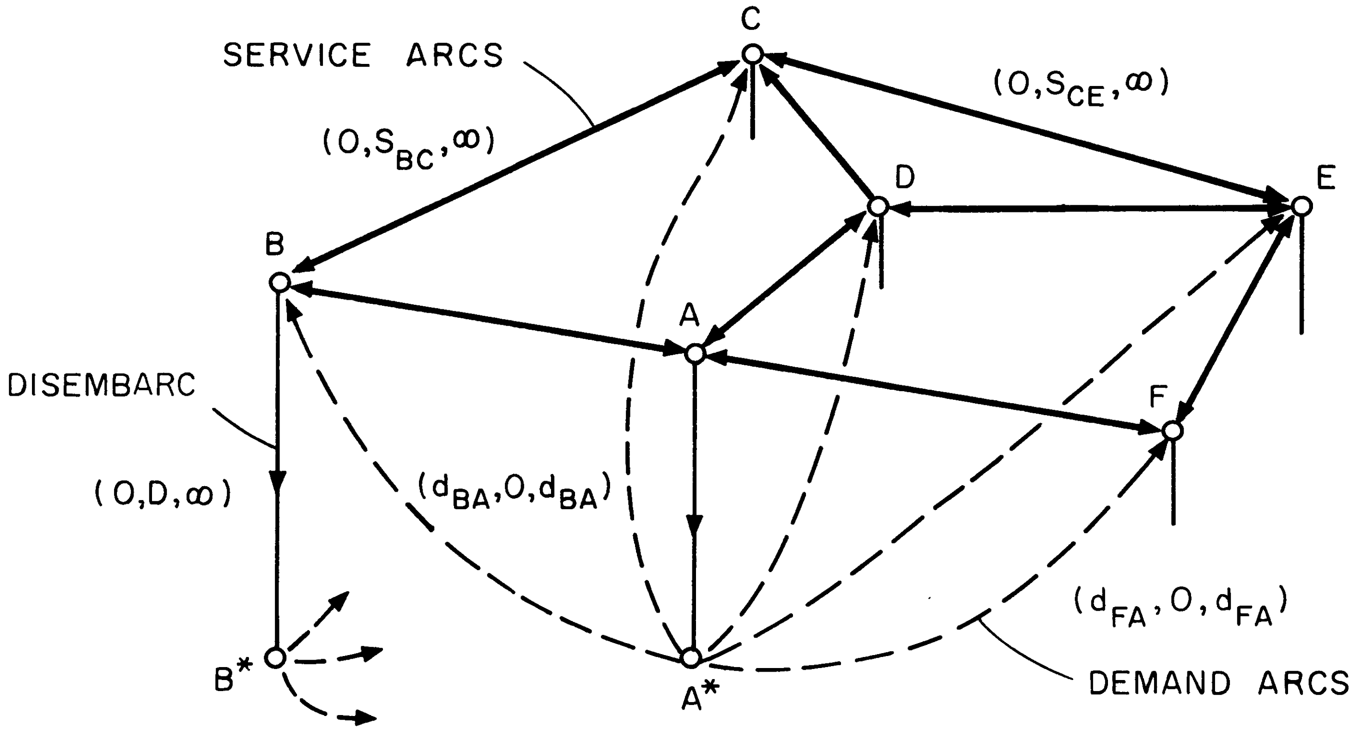
This is recognizable as a special case of the general linear program where $a_{ij} = 0, \pm 1$, and is called the transportation or assignment problem. It is widely applicable to a variety of transportation or scheduling processes, particularly since the size of the Out of Kilter algorithm and its integer solutions allow practical problems to be solved. If posed as a linear program, there would be 4500 variables with 9000 arc equations for the upper and lower constraints and 1500 node equations.(i.e. a matrix of 10500 rows and 4500 columns). Since most of the a_{ij} matrix entries are zeroes (matrix densities for this type of problem are typically 10 percent), the labelling technique used by Ford and Fulkerson is far more efficient than any variant of the Simplex technique. If l_{ij} and u_{ij} are integers, we are assured that any feasible solution for X will also be integer.

Description of the Model for Passenger Flow

To use this technique, it is necessary to construct a graph or network as a model or representation of the passenger flow problem. Figure 3 shows a simplified network representation of the model.

The subgraph of "city" nodes $\left[A, B, C, \ldots\right]$ and "service" arcs $\left[AB, BA, AD, DA, \ldots\right]$ represents the geographic or service network of cities A, B, etc. and non-stop services operated between cities AB, BA, etc. . If service between cities A and B is to be operated non-stop, then a pair of directed "service arcs" AB and BA exist in the service network. In Figure 3 this pair of arcs is represented as a solid line with both directions indicated by arrows. Each arc has its cost (c_{ij}) value set to s_{ij}, the distance in miles between the city pair. Every service arc has $l_{ij} = 0$, and $u_{ij} = \infty$. Thus, there are no capacity constraints on the flow in service arcs, and the value of the flow, x_{ij}, will represent the number of passengers/day wishing to use this service.

The arcs $\left[AA^*, BB^*, \text{etc.}\right]$ are another subset of arcs called "disembarcs". Nodes $\left[A^*, B^*, C^*, \ldots\right]$ are called



FIRURE 3(a) NETWORK FOR PASSENGER FLOW SOLUTION

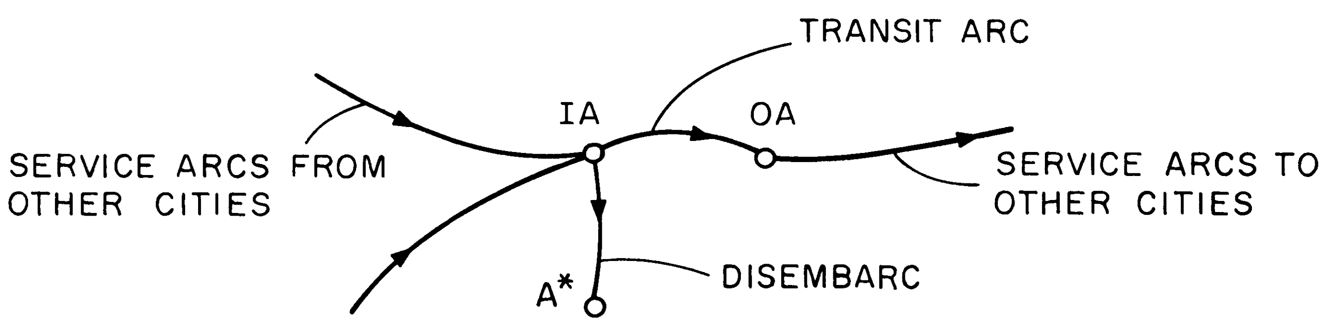


FIGURE 3(b) MODIFICATION OF CITY NODES

station nodes. There is one disembarc for each city. Their values of l_{ij} and u_{ij} are 0 and ∞ respectively as for the service arcs, and the cost for these arcs is D, the diameter of the service network. A diameter is the longest track or elementary path in a network, and this value is placed upon these arcs to prevent flows from disembarking at an intermediate station and travelling via demand arcs to their destinations. The value of the flow in a disembarc represents the number of people arriving at that station per day. It will equal the sum of the associated column of the D matrix.

From every station node [A*, B*, C*, ...] there is a subset of arcs called "demand" arcs. For example, from node A*, a demand arc exists going to all city nodes [B, C, D, ...] . The cost on these arcs is zero, and both l_{ij} and u_{ij} are set to a value representing the daily demand in passengers/day between city B and all other cities. These demand arcs ensure that a flow x_{ij} equal to the demand d_{ij} exists in the demand arc, and necessitates a return flow via the shortest path through the service network. For example, the demand arc A*E in Figure 3 causes a flow in the service network back to A and A* via either EDA or EFA whichever is shortest. The node conservation constraints cause all flows in the complete graph to be circulations, or flows in a circuit. For an n-city problem, there will be n x n-1 demand arcs. If the demand is symmetrical, some modifications can be made to avoid repetition of demands. In this case, there are only $\frac{n \times n-1}{2}$ demand arcs.

At this point, the multi-commodity aspect of the problem is encountered. Every x_{ij} flow into a given city must be considered as one commodity, x^n_{ij}, to prevent the flow x_{ij} along the shortest path in the service network from being exactly cancelled by the symmetrical flow, x_{ji}, along the identical path in reverse order. A modification of the labelling technique in OKF was made to remove the reverse labelling. In this way, an x_{ij} value, once placed in the network, could not be removed, and the return flow, x_{ji}, uses the symmetrical forward path along the other member of the service arc pair.

Since there are no capacity constraints on service arcs in this problem, the multi-commodity aspect can be handled efficiently within one run of the OKF code by solving for each city sequentially. The "Alter" option of the coding was used to impress new demands for the flow into the next city.

The complete solution for a 50 city case takes around 10 minutes on the MIT IBM 7094 while in CTSS operation. (Compatible Time Sharing System).

Information Available from the Model

This modified algorithmic process will minimize the total flow costs over the complete graph:

$$\text{i.e. Minimize } \left[Z = \sum c_{ij}\, x_{ij} = \sum_{\substack{\text{service}\\ \text{arcs}}} s_{ij}\, x_{ij} + \sum_{\text{disembarcs}} x_{ij}\, D \right]$$

But, since D = constant, and for any given demand the x_{ij} in the disembarcs is fixed equal to the column sums in the D matrix, i.e., the total number of originating passengers per day, P.

$$\therefore \quad \text{Min } (Z) = \underset{\substack{\text{service}\\ \text{arcs}}}{\text{Min}} \left[\sum s_{ij}\, x_{ij} \right] + P.D$$

$$= PM_{min.} + P.D$$

The constant P.D is readily calculated and subtracted from Z to get PM_{min}. We are minimizing total passenger miles (PM) in the network, and every individual passenger will be travelling his shortest route since there are no arc capacity constraints.

The average passenger trip distance is then:

$$\bar{D}_p = \frac{PM}{P}$$

By a simple trick of splitting the nodes $\left[A, B, C \ldots \right]$ into two parts separated by a "transit" arc, as indicated in Figure 3b, further information can be obtained. For example, node A becomes two nodes: IA which receives all service arcs from other cities, and OA which starts all service arcs out of city A. The transit arc $\{IA, OA\}$ has a flow which is the number of people passing through the station on their trips to other destinations.

The number of passenger departures per day, PD, is obtained by summing x_{ij} in the transit arcs and adding P.

$$PD = \sum_{\substack{\text{transit} \\ \text{arcs}}} x_{ij} + P$$

From this we can obtain the average hop or non-stop distance for both passengers and vehicles:

$$\overline{D}_H = \frac{PM}{PD}$$

As well, the frequency distribution of hops can be obtained. If we categorize service arc distances, then the sum of x_{ij} for arcs in each category are an indicator of frequency.

The average number of hops/passenger is

$$\overline{H} = \frac{PD}{P}$$

The most detailed information available from each solution is X itself. For the service network $X = (x_{ij}$ for all ij in network), is precisely the F matrix which gives the number of passengers/day using each service.

It is possible to obtain the indirect routings and the number of passengers per day on them. This has not been carried out as yet. That information could be useful in constructing flights consisting of a series of flight segments by the same vehicle at a later stage in the schedule construction process.

The Network Models for the VTOL Airbus System

For a system serving the 50 shopping points shown in Figure 1, a model was constructed. There are 2450 possible demand arcs which obtain d_{ij} from the demand generation program. There are 50 disembarcs, and 50 transit arcs, and somewhere between 200-500 service arcs. The distance costs, s_{ij}, are the great circle or airline distances used in the demand program. The value of D was set at 1000 miles.

Several solutions were obtained for both D_1 and D_2 demand matrices, using different service networks. The service networks were selected on two criteria. The first was geographic where each station was connected to two of its closest neighbors by "basic" arcs. These basic arcs were members of every service network.

The second criterion was either demand, d_{ij}, or passenger flow, x_{ij}. Table 4 describes the sequence of runs for the second demand, and indicates how the various service networks were selected. A strategy of including arcs of lesser O and D demand d_{ij} was followed in the first four runs, and the frequency of service pattern determined as explained in the next section. Adding more and more non-stop services dilutes the service to the point where a small number of

TABLE 4. COMPUTER RUNS FOR SECOND DEMAND DATA

Run	Description of Service Network	$\bar{D}_H$ Average Hop Miles	Z Total Pax-miles per day	No. of Service Arcs	PD No. of Daily Pax. Departures
1	Basic network - two closest stations	48.5	-	208	-
2	Basic, + all arcs where $d_{ij} \geq$ 300 pax./day	58.4	11.47×10^6	250	196,371
3	Basic, + all arcs where $d_{ij} \geq$ 200 pax./day	74.7	11.32×10^6	332	151,414
4	Basic, + all arcs where $d_{ij} \geq$ 100 pax./day	-	11.19×10^6	534	-
5	Run 4, minus non-basic arcs whose $x_{ij} \leq$ 100	87.4	11.22×10^6	410	128,249
6	Run 5, minus non-basic whose $x_{ij} \leq$ 150	83.3	11.25×10^6	360	135,134
7	Run 6, minus non-basic whose $x_{ij} \leq$ 250	79.1	11.28×10^6	316	142,454

passengers per day were using various lesser services. The strategy then became that of dropping service on these low density routes and asking these passengers to proceed indirectly via routes which enjoyed higher passenger flows. Basic service arcs were always retained so that no station could be isolated. Passenger flows on basic arcs is often very low, and can be used to consider dropping the station from the system.

TABLE 5 - THE PASSENGER FLOW MATRIX (F) AND THE FREQUENCY PATTERN MATRIX (N)

SECOND DEMAND, RUN 7

F MATRIX DAILY PASSENGERS - BOTH WAYS

	DCA	DUL	BAL	BOS	BOC	MAN	WOR	NBD	PVD	WIL	PHL	TRE	REA	LAN	ALL	HAR	WBA	SPR	PIT	BDL	JFK	LGA	JRB	EWR	BDR
DCA		126																			659	766			
DUL	2																							436	
BAL																									
BOS					1087																700	1135			
BOC				11			419		528												594	2324		830	
MAN																									
WOR					5													530				1860			
NBD									278																
PVD					6			3													627	1533			
WIL											1206											835			
PHL										13				566							968	1798		614	
TRE													90									538	219		
REA												1		1703	1569	288	899					629			
LAN											6		18									755			
ALL													16									751		751	
HAR													3									638		684	
WBA													9									409			
SPR							6												366	585		1531			
PIT																		4							
BDL																		6							
JFK	7			7	6				7		10											1959	182	602	
LGA	8			12	24		19		16	9	18	6	7	8	8	7	5	16			20		1796		1374
JRB												3									2	18		878	
EWR		5			9						7				8	7					7		9		
BDR																						14			
HVN																						20			12
WBY																									
GON								11	6													11			
HFD				7	6				2											12		14		7	
ACY																							7		
BMR			12							7	17			16							6	19		8	
WAS	18	8	3								15										6	10		10	
MIT					9				12		14										10	14			
TBO											8												2		
NYC				21							9				6						3	32	4	14	
ORF																						10			
RIC	7																				5				
PPA	14	17	15	12	7						24	5	12					10			16	29	18	11	
PHF																					8				
YRK														6		7						7			
AVP																	7								
SGC											8											8			
NWK																									9
ISP																									
EQU																						6			
EHM																									
PWM																									
FIT					1	4	8												1						
BTN					2			9																	
LAW				14		1																			

	HVN	WBY	GON	HFD	ACY	BMR	WAS	MIT	TBO	NYC	ORF	RIC	PPA	PHF	YRK	AVP	SGC	NWK	ISP	EQU	EHM	PWM	FIT	BTN	LAW
DCA							1799					620	1374												
DUL							752						1687												
BAL						1181	280						1445												
BOS				667						2064			1111												1301
BOC				583				827					607										65	112	
MAN																							399		49
WOR																							751		
NBD			1083																					854	
PVD			587	178				1172																	
WIL						633																			
PHL						1616	1420	1351	747	899			2373				725								
TRE													423												
REA													1106												
LAN						1552									524										
ALL										593															
HAR															692										
WBA																685									
SPR													936												
PIT																							39		
BDL				1188																					
JFK						570	522	973		259		484	1519	794											
LGA	2000		1012	1345		1886	983	1331		3172	942		2869		604		763			581					
JRB					617				120	340			1714												
EWR				646		715	960			1307			1029												
BDR	1197																	847							
HVN		851	572	1595															622						
WBY	9			161																					
GON	6												758								343				
HFD	16	2											805												
ACY													401												
BMR							1664			763	514	745	1360		720										
WAS						17		509		733	503		1424												
MIT							6				548		1613				261		1683						
TBO										2093			867			824									
NYC						8	8		21				957												
ORF						6	6	6					1158	70											
RIC						8							1094	616											
PPA			8	9	5	14	15	17	9	10	12	11		850			738			575					
PHF											1	7	9												
YRK						8																			
AVP									9																
SGC								3					8					1190							
NWK																	12								
ISP	7							17												288					
EQU													6						3		672				
EHM			4																	7					
PWM																									518
FIT																									
BTN																									
LAW																						6			

N MATRIX DAILY FREQUENCY - ONE WAY

Typical Results

Results from the final run for the second demand are given in detail in Table 5. Entries in the upper half of the matrix are x_{ij} = passengers per day using service ij.

Other interesting results are given below

Total pax-miles/day = $Z = 11.28 \times 10^6$

Total Passenger Trips/day = P = 76000

Total number of Passenger Departures = PD = 142,454

Average Passenger Trip Distance = $\overline{D}_p$ = 148.5 miles

Average Hop Distance = $\overline{D}_H$ = 79.1 miles

Average No. of Hops/Passenger = 1.91

The distribution of hop distances is shown in Figure 4, compared to the distribution of city pair distances for the 50 stopping points.

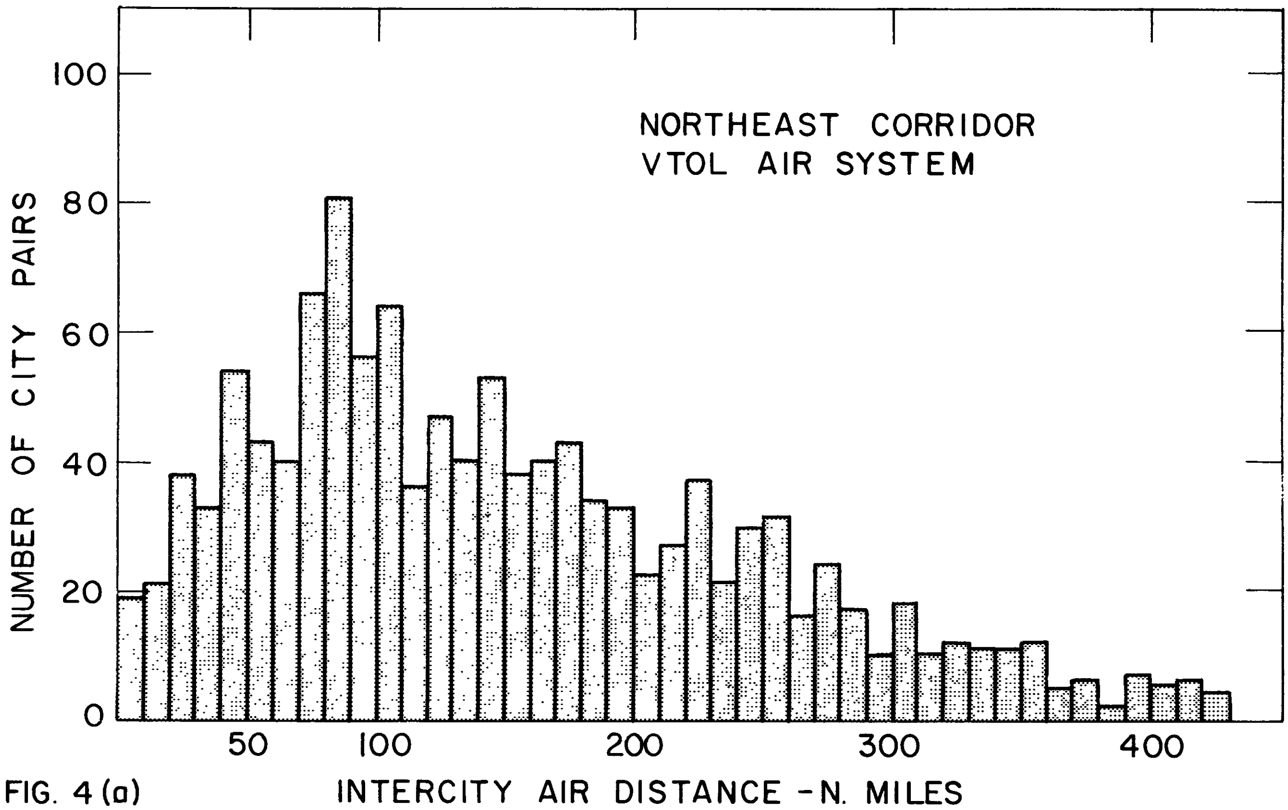


FIG. 4 (a)

DISTRIBUTION OF INTERCITY DISTANCES (ALL CITY PAIRS)

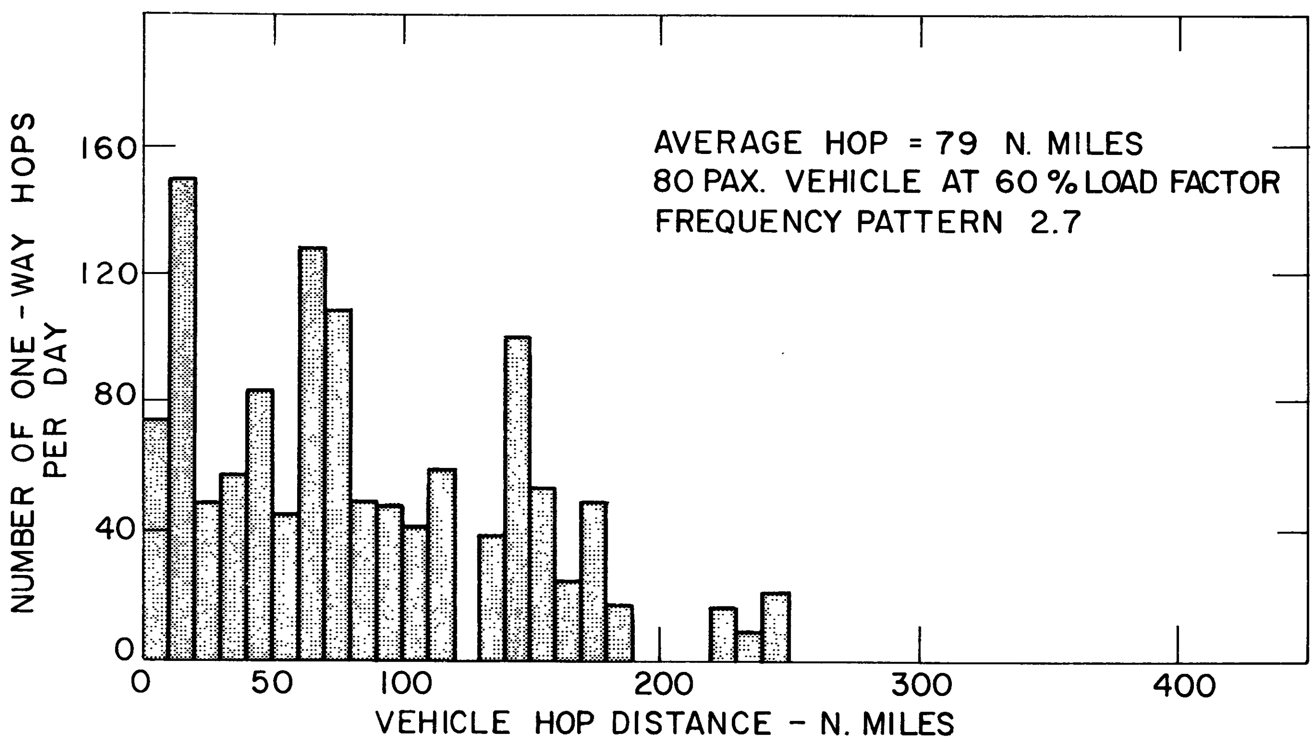


FIG. 4 (b) DISTRIBUTION OF VEHICLE FLIGHT HOPS

FIG. 4

IV. DETERMINATION OF THE FREQUENCY PATTERN

Given the expected number of passengers/day using a given route, an estimate of the desirable number of services or direct flights/day can be obtained using vehicle seat size and an average load factor established by planning policy. If we let N_{ij} be the number of flights/day;

$$N_{ij} = \left\langle \frac{x_{ij}}{S \cdot LF} \right\rangle$$

where x_{ij} = daily passengers from i to j - one way

S = vehicle seat size

LF = desired average load factor

$\langle\ \rangle$ signifies rounding off to next highest integer.

At the present time, it has been assumed that there is only one vehicle size (80 passengers), and that for planning purposes an average system load factor which should be achievable is 60%. Other combinations of vehicle size should be studied since within the present models there are indications that a smaller vehicle on the low density routes should be used to increase daily frequencies of service.

The average load factor of 60% is chosen to allow for monthly and weekly cycles of demand, and to account for the fact that daily demand is a probabilistic variate from day to day. When a fixed seating capacity is matched against the average of a probabilistic quantity, a capacity margin above the average load is required to ensure that the above average loads can be carried.

The time of day cyclic variations can be accounted for in two ways: allowing load factors to vary with time of day, and by bunching departure times at the peak hours. Both methods are discussed in the next section.

It is assumed that a daily schedule will be established for the period of the demand estimate. If more detailed information were available regarding weekly variations in demand, considerations could be given to such things as a semi-weekly schedule or special schedules for Saturday, etc. Typical cycles of demand are shown in Figure 5. Similar information can be generated and estimated by the management information system for the Airbus system.

It has also been assumed that there is no information on competitive or marketing considerations which would differentiate one route from another. Normal domestic airline competition causes frequencies to be added by competing airlines to the point where marginal revenues tend to equal marginal costs. As such, a variation in breakeven load factors with length of haul arising from the differences between fare and cost structures causes varying market load factors in competitive markets. It has been assumed here that fares are proportional to cost, causing equal breakeven load factors on all routes, and allowing the planned load factor to remain constant for all services. In this manner, the allocation of seats/day to a given service is directly proportional to the estimated number of passengers. Given other situations such as the present airline system, different assumptions and allocations of seats/day would be made using suitable marketing information. No such information exists for this study.

The frequency pattern, or N matrix, corresponding to the F matrix is given in Table 5 below the diagonal. It assumes an 80 passenger vehicle at 60% load factor, and is symmetric. The entries are the number of one-way flights/day for each service. It is interesting to note at this point that the N matrix of Table 5 has 2996 flights/day which is 2-3 times as big as the largest airline schedules in existence today.

A frequency distribution of the number of one-way non-stop services/day is given in Figure 6 for this N matrix. The average value is 9.5 non-stop flights per day, and the

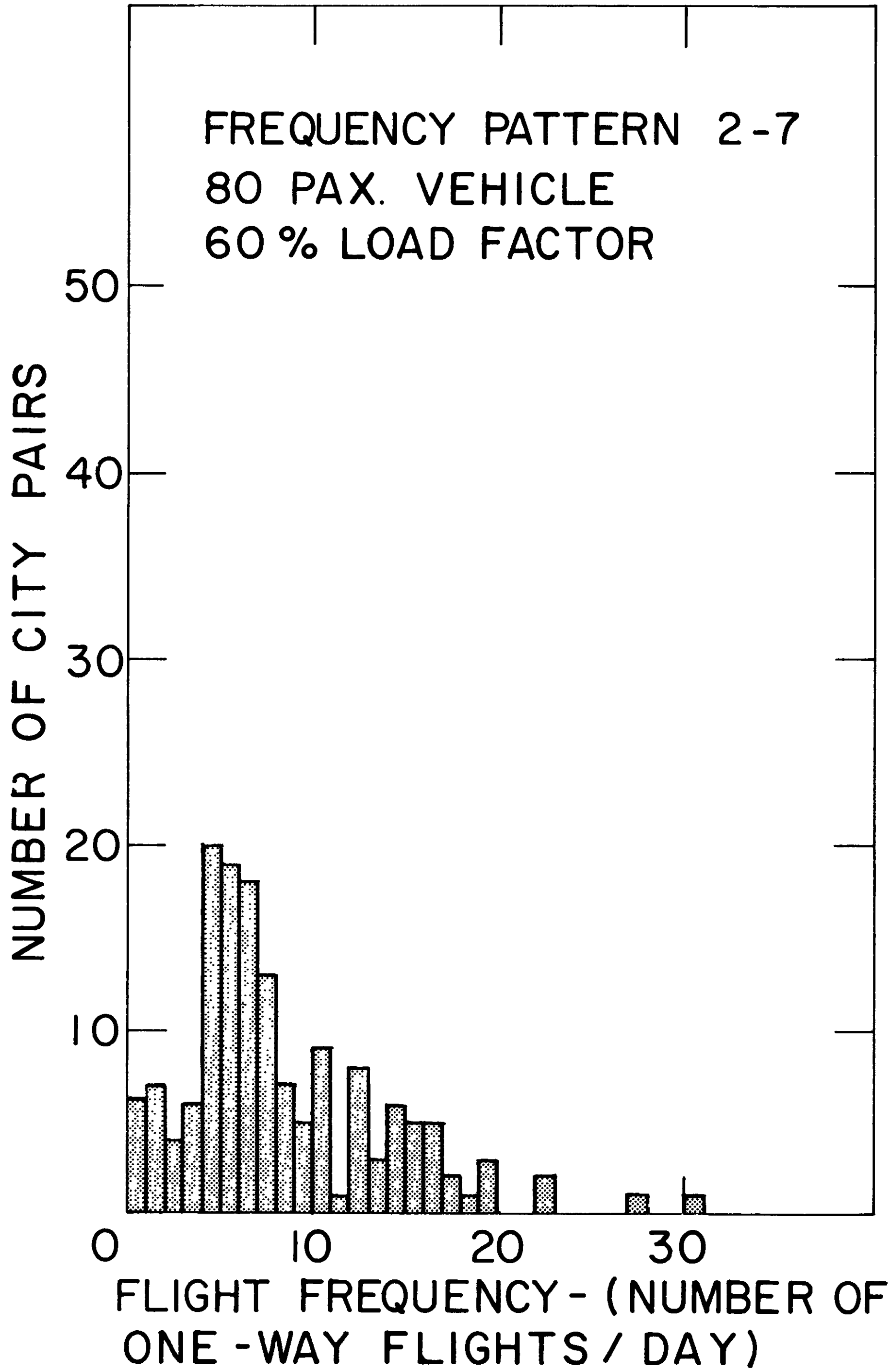


FIGURE 6 DISTRIBUTION OF FLIGHT FREQUENCY (NON-STOPS)

distribution indicates that there are many routes below this value. There are, however, a significant amount of one-stop, two-stop, etc. indirect services on most routes which should also be considered in determining the total frequency of service.

The routes below 5 frequencies per day indicated in the distribution are all "basic" services retained to keep certain cities in the system. Also, examination of the total system effect of dropping such cities can be easily made.

The number of operations per day for each station can be obtained by summing the row and column of a complete N matrix. The row sum represents the number of departures, and equals the column sum which is the number of arrivals. Totals for run 7 are given in Table 2 for all cities. The largest station for this pattern is Laguardia airport with 798 operations per day. This would indicate use of larger aircraft in Laguardia Service, or the establishment of another terminal in the New York area. Similar considerations would apply to the downtown Philadelphia site which has 646 operations per day. With the computerized methods used in this study, new sites would be chosen in such areas, and a new passenger flow pattern determined using a revised estimate of demands between all stations and the new sites. The N matrix or frequency pattern, and the new number of operations/day per station are then quickly tabulated.

V. TIMETABLE CONSTRUCTION

Having determined a frequency pattern for all non-stop services, the next step in constructing a timetable or schedule plan is to assign departure times for each of the N_{ij} services on every route ij. Given a departure time for a flight from i to j, and knowledge of the trip duration or block time, the arrival time at j is determined. The set of departure and arrival times, properly ordered for every station, constitutes a timetable describing in explicit detail the transportation system.

A computer program has been written to construct an initial timetable given as input at this point three sets of data: 1) the N matrix, or frequency pattern giving N_{ij}; 2) the ΔT matrix describing block times of ij; 3) data describing the daily variation in demand, $d_{ij}(t)$, for every city pair.

Determining Suitable Departure Times

In the absence of detailed information about daily variations in demand for the hypothetical 1980 Airbus System, two demand variations were assumed. A flat distribution from 0600 to 2400 hours, and an extremely peaked distribution descriptive of Eastern airlines shuttle demand on a Friday. These were considered as extremes, and that the daily variation would lie somewhere between them.

These daily patterns were chosen after examining a variety of patterns from various sources. The daily traffic patterns for Northeast airlines, for all days of the week and various months of the year were available. Traffic patterns reflect passenger demand modified by the airline schedule, and the peaking was much less severe than the EAL shuttle pattern used. There were wide variations in patterns at different times, on different routes, and from day to day. The number of aircraft operations per hour at various Northeast Corridor airports was examined for various periods to see the daily pattern as reflected by Airline schedules, etc. Again the patterns were less peaked.

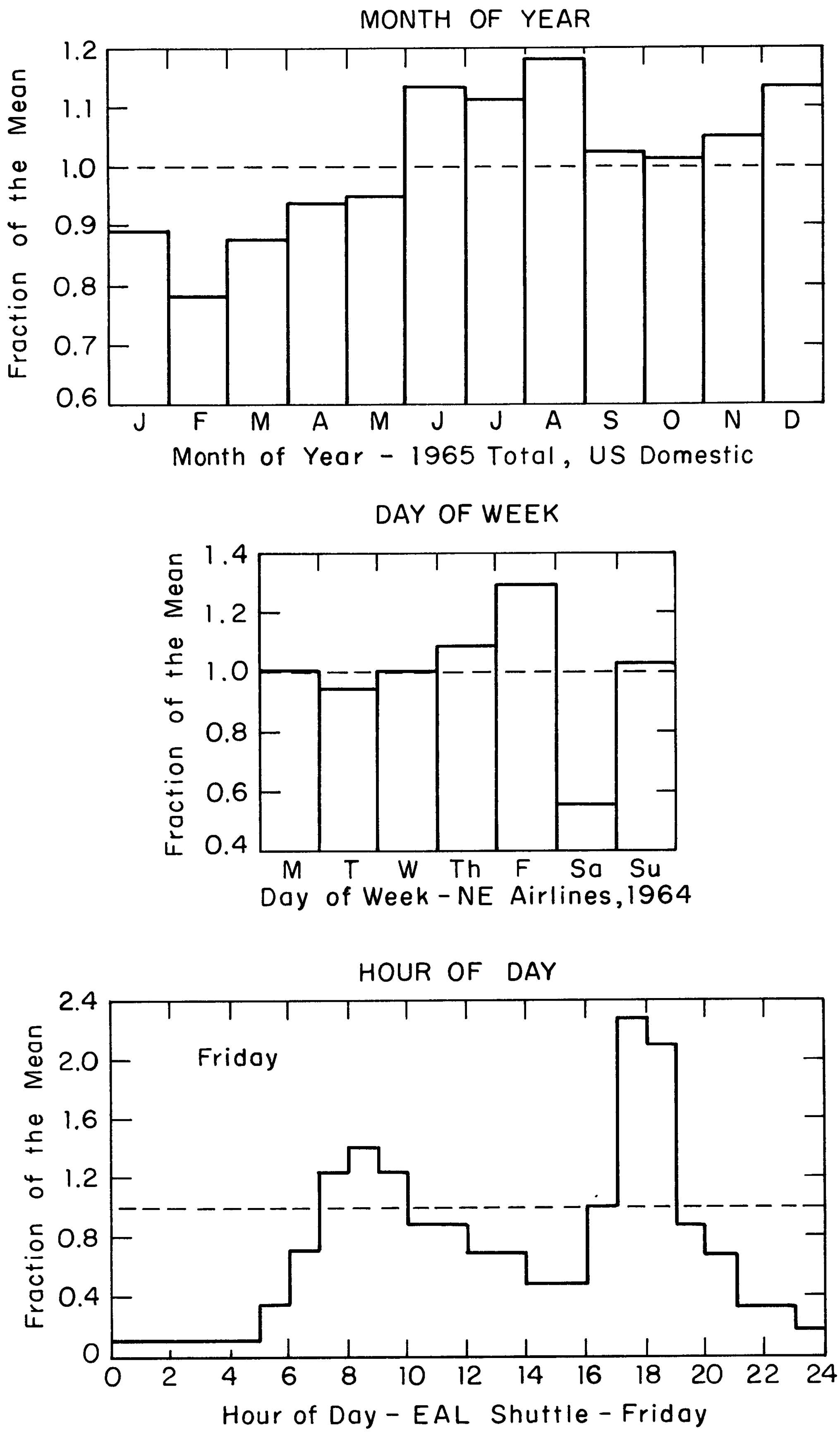


FIGURE 5 TYPICAL AIRLINE DEMAND VARIATIONS

Finally, detailed passenger and aircraft information for one day's operations at New York JFK Airport were examined to see the hourly variations in passengers and number of seats for domestic services. Load factors were substantially higher during peak periods, since the passenger distribution was more peaked than the aircraft distribution.

It is not possible or desirable to present all the information on daily variations examined. A sample of typical airline traffic variations is shown in Figure 5 .

The method of choosing departure times for N_{ij} flights in ij service is explained by Figure 7. The two distributions of the arrival rate of passengers (or of the probability density of passenger demand) are shown in the upper portion of Figure 7. Below them are the cumulative probabilities of demand obtained by integrating the probability density distributions or arrival rates. It goes from zero to 100%, and represents the cumulative number of passengers arriving during the day. The uniform arrival rate gives a straight line cumulative from 0600 to 2400, while the peaking shows much steeper slopes during the peak periods.

The rationale first used to select departure times was to divide the daily load on a given route equally amongst N_{ij} departures by dividing the vertical axis of the cumulative distribution into N_{ij} equal segments. The departure times could then be found by reading the corresponding time from the horizontal axis. However, because of the optimization process described in the next section, this was changed such that a range of times was selected for each departure in a similar manner. Thus, the vertical axis was divided into $2N_{ij}-1$ parts so that N_{ij} departure ranges could be selected. The departure time was placed in the center of each range to form an initial timetable.

This is shown in Figure 7 for both distributions when $N_{ij} = 6$. The vertical axis is divided into 13 segments, and the departure ranges are shown by the shaded bands. For the uniform or flat distribution, the departures are equally spaced. For the peaked distribution, the departure times tend to be bunched at the peak hours, when their ranges are also much reduced. There is always a gap between departure ranges such that two successive

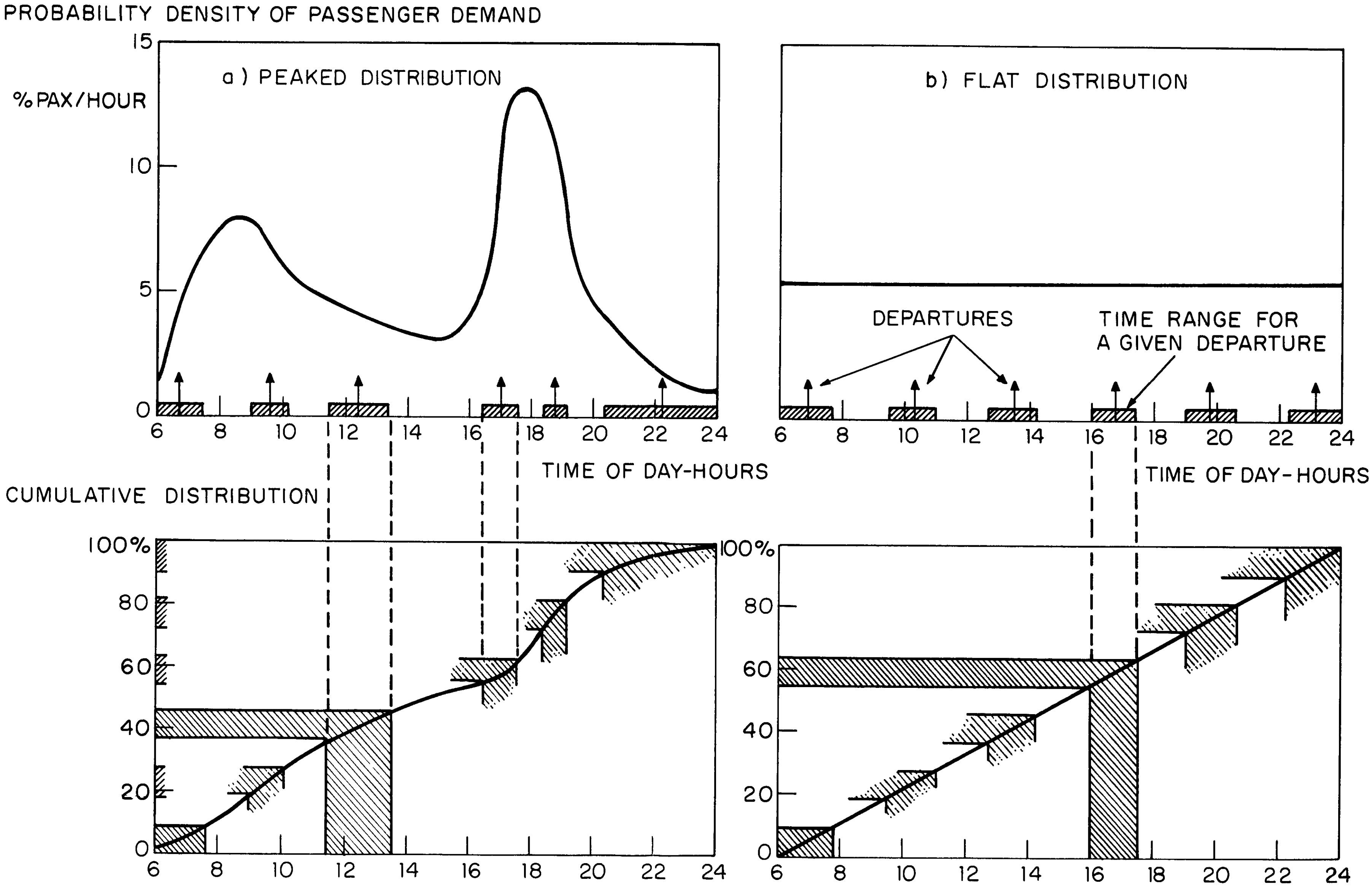


FIGURE 7 METHOD OF DETERMINING DEPARTURE TIMES
—EXAMPLE FOR 6 DEPARTURES / DAY

departures for the same destination will always be separated even when they are moved to the closest end points of their ranges.

The initial departure times chosen by this method are symmetrical in that flights leave both i and j for j and i respectively at the same time. Furthermore, there will be departures at precisely these same times for every city pair with the same number of daily flights, since the daily variation in demand is used for all city pairs. This symmetry is partially destroyed during the optimization process of the next section.

There has been no consideration of continuing, through flights at this point. They would be constructed after seeing the connectivity of the flight hops after the optimization process of the next section. Similarly, there has been no consideration of interconnections between flights, or between other modes of transportation. The possibility of competition (a critical factor in choosing times for airline schedules) has not been considered here. All these considerations would be introduced at a later stage if pertinent information were available. Notice that the departure times are chosen from the same daily variation for all routes, representative of O and D demand, not indirect demand through a station.

The two daily variations in demand, and the resulting choices of times represent two extremes of the problem of peaking. The choice of departure times from this method also represents two philosophies of matching service to these demand variations. It is possible, for example, to choose times uniformly throughout the day, and allow the load factor variation to handle the peaking. An average load factor of 45% would give peak load factors of 100% during the 5-6 pm peak for example. This approach avoids bunching of departures in order to maximize utilization, and accepts lower average load factors. However, the low load factors on off peak flights causes proponents of this approach to comment on sensitivity of loads to timings, and to move flights towards peak times whenever possible.

The second philosophy is to maintain load factors on individual flights, and to bunch departures at peak times. Load factors are higher at the expense of aircraft utilization, and the lower utilizations cause movement of flights away from optimum market times in order to make connections which ensure better vehicle usage.

The results in either case begin to resemble each other, and it is expected that the two results obtained here will bracket a reasonable schedule.

Timetable Construction Program

A computer program was written to construct an initial timetable for the 50 stopping points of the Airbus system. It accepted information on the cumulative distribution of passenger demand, the block times for a given vehicle on each city pair, and the N matrix or frequency pattern, and it gave as output an ordered list of events (arrivals and departures) for each station, as well as punched output suitable for the program of the next section. At every station, the list of arrivals and departures indicated the other city involved and gave the event time to the nearest hundredth of an hour. The program constructed the timetable of 2996 flights in about 4 minutes of running on a time-shared IBM 7094.

VI. OPTIMIZATION OF VEHICLE UTILIZATION

A range of departure times for each service was chosen so that departure times could be varied to allow better connections for vehicles and passengers. In this way improved vehicle utilization could be obtained, which has a strong effect on direct operating cost. Depreciation costs for a typical 3-4 million dollar Airbus vehicle are about 30% of the DOC. Maximizing utilization is equivalent to either minimizing ground time, or the number of aircraft in the fleet required for a given timetable. This may be seen from the following:

For a given schedule, the total amount of block time is fixed (assuming 1 aircraft type). Let this be called BT, for the daily number of hours flown in the schedule; Average Vehicle utilization, $\overline{U}$, in terms of average hours/day per vehicle, given that fleet size is NF is simply;

$$\overline{U} = \frac{BT}{NF}$$

Therefore, since BT is constant U is maximized when NF is minimized

$$\max \{U\} \Rightarrow \min \{NF\}$$

However, if there are NF aircraft in the fleet, there are $24 \times (NF)$ fleet - hours per day. Aircraft in the fleet are either flying or on the ground. If we call the total fleet ground time, GT, then

$$24. (NF) = BT + GT$$

Since BT is constant for a given schedule, the minimization of NF is equivalent to minimizing GT. GT can be further divided into two parts: the load-unload time necessary for all flights, which is a constant, ST; and a ground waiting or waste time, WT, where aircraft are available for service, but are not being used. WT is the component which can be minimized.

In this section, a heuristic algorithm will be described which minimizes NF given a schedule and a description of allowable ranges for every departure time in that schedule. It does not achieve a true optimum. It is one of three developed at MIT in the last year which have the simple capability of reducing NF with varying degrees of success. Obtaining the true optimum for such sequencing problems seems to be beyond the state of the art for operations research at the present time. The use of similar methods, or simulations on "job-shop" problems is typical of methods used on such sequencing problems.

A basic assumption implicit in the statement of this problem is that services can be varied within some range without any change in the amount of revenue or traffic associated with the flight. In airline practice, where competition may exist, this range can be very small. However, in other cases, the airline marketing analysis often associates a broad range of times with the service. The ranges are chosen rather arbitrarily in this study because of the lack of any detailed data. The algorithm will accept any well defined range for every service. Departure times can be fixed by having the upper and lower limits of the range coincide. Some latitude in departure times is necessary of course for the optimization to be able to operate.

Enumerating the Vehicles Required by a Given Timetable

If we add a minimum turnaround time to every arrival time, we have a "ready" time for every arrival. This has been done by the timetable construction program, and the examples shown in this report of various timetables actually use "ready" times to describe arrivals. The actual arrivals occur 6 minutes earlier and a slight modification is required to count the true number of aircraft at the station for any given time. The minimum turnaround time was taken as 6 minutes based on the analysis of turnaround times in the previous report, Reference 3. It consists of an average load-unload-refuel time of 5 minutes, plus 1 minute margin for reliability and describes a transit or through flight operation. Engines are not necessarily stopped.

The timetable describes for every station a list of time ordered events of two types: first, a "ready time" event when an aircraft arriving from another station becomes ready or available for service; secondly, a departure event for services to other stations. Figure 8 shows such a typical event sequence, E. If we define

Figure 8 - METHOD OF COUNTING VEHICLES AT A STATION

D = Departure NA = Number of aircraft at station after each event

R = Arrival ready

TIME	E	NA_1	NA_2
		Put NAC = 100	Put NAC = 3
0715	D	99	2
0730	D	98	1
0800	D	97*	0
0855	R	98	1
0930	R	99	2
1100	D	98	1
1215	R	99	2
1400	D	98	1
1705	R	99	2
1730	D	98	1
1920	D	97*	0
2040	R	98	1
2100	D	97*	0
2245	R	98	1
2305	R	99	2
2330	R	100	3
Number of aircraft overnight		NAC = 100	NAC = 3
Smallest number in NA sequence		= 97	= 0

NA to be the number of aircraft on the ground after each event, the NA sequence consists of numbers which differ by unity. For a departure, one is subtracted from the previous NA, and for an arrival "ready", one is added to the present event's NA. We may start the NA sequence with any large number, NAC, which represents the number of vehicles which will "overnight" at the station. Figure 8 uses NAC = 100 in starting the sequence in the column NA_1.

If we find the smallest member in the NA_1 sequence, and subtract it from every member of the sequence, we get the NA_2 sequence which will have a number of zeroes (at least one) appearing somewhere in the sequence. This sequence represents the minimum number of vehicles required to carry out the timetable at this station. The total minimum fleet, NF, can be counted by adding NAC for every station; i.e., the total number of aircraft overnighting at all stations.

$$NF = \sum_{i} NAC$$

This assumes that there is some period during the night at which the total fleet is on the ground. This is usually possible for short haul passenger transport schedules.

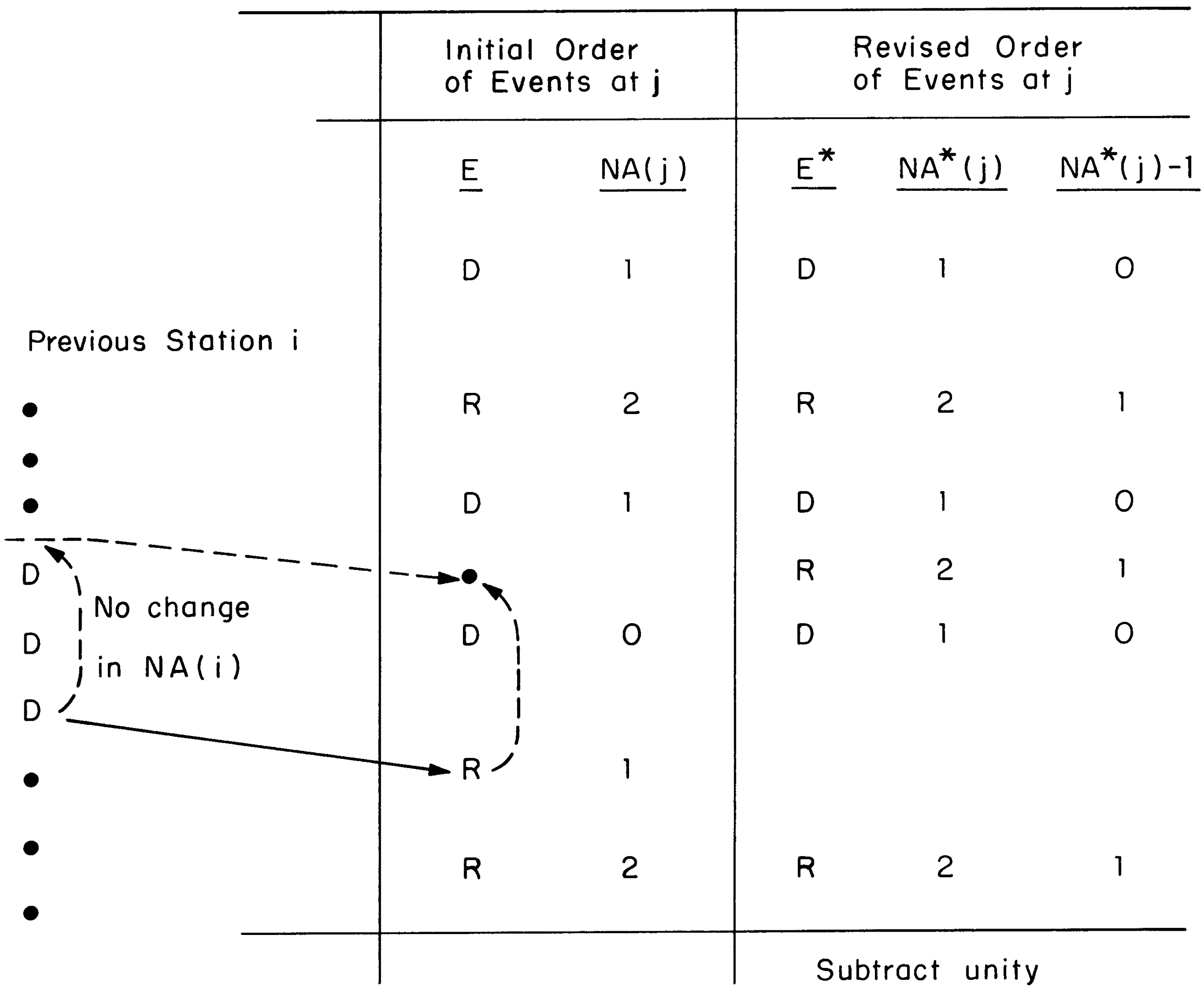
The connections or "turns" which the vehicles on the ground make between incoming and outgoing services is not explicitly stated. If there is only one aircraft on the ground before a departure, then it must be used on the departure service. However, if there are two or more, any one of them can be used since they are all ready for service.

If we adopt a strategy for connections of "last in - first out", then we can show that the NA2 sequence is truly minimal. For if there is one (or more) aircraft on the ground at all times, it is never used in any service and is unnecessary (except perhaps as a spare or "cover" aircraft for schedule reliability). To use the last vehicle, a zero must appear after a departure at least once in the minimal NA sequence. Of course, NA cannot contain a negative number since it would represent a negative number of vehicles on the ground. This counting logic is well known to schedulers, and even has been discovered by more sophisticated methods of operations research! It will not be proven here.

A Logical Method for Reducing NF and WT

If we are given a timetable, and a corresponding set of minimal NA sequences, we may be able to reduce NAC for any station j by interchanging departure and arrival events such as to increase the zero values in the NA (j) sequence. If it is possible to increase all the zeroes in the NA(j) sequence by unity, then we have a new sequence of events, E*, whose $NA^*(j)$ sequence is no longer minimal. The new minimal NA*(j) sequence is obtained by subtracting unity from every member of the sequence. The last member of the sequence is NAC*(j) which is thereby reduced by one. Providing the interchange of events a station j did not increase NAC at the previous stations (i) and downstream stations (k), then the fleet size NF will have been decreased by unity.

An example of this logic can be given with the aid of Figure 9. For the original sequence of events at station j, a zero appears after the third departure. It is possible to change this zero to unity in two ways: 1) Move the corresponding departure after any of the following arrivals - provided the departure remains within its defined range of times; 2) Move a later arrival ahead of the zero departure - provided the arrival remains within the range of times associated



NAC(i) = constant NAC(j) = 2 $NAC^*(j) = 1$

$\therefore NF^* = NF - 1$

FIGURE 9 EXAMPLE OF REDUCING NAC AND NF

with its flight.

In the example, the next arrival has been moved ahead of the zero departure, and there has been no change in the NA(i) sequence, and therefore NAC(i) remains constant. The revised NA*(j) sequence is no longer minimal. Unity can be subtracted, giving a new minimal NA*(j) sequence of 01010..., and reducing NAC*(j) to unity. NF* is also reduced by unity.

The same sequence of events could have been obtained by moving the zero departure after the next arrival provided the move is within the departure range, and that any changes at the corresponding arrival station, k, did not increase NAC(k). Although the sequence is the same, the times associated with the departure and arrival are different in the two alternatives.

An absolute minimal sequence consists of events RDRDR... RD at a station with the corresponding NA sequence 1010...01010. In this case every arrival is connected to the next departure, and NAC is zero. There is a complementary sequence, DRDR...DR, with NA sequence 0101...0101, where NAC is unity, and the overnight vehicle is used for the first event in the morning which is a departure. Notice that the law of conservation of vehicles for any schedule plan states that the number of arrivals equals the number of departures. A corollary of this is that the number of events at every station is an even number.

It may appear that WT, the ground waiting time, can be reduced for any particular station j even when that station has the absolute minimal sequence. Figure 10 shows such a case where an arrival at 1230 pm at station j connects to a departure at 1330. The ranges of flight times would allow the flight to arrive and be ready as late as 1245, and depart as early as 1300. This is an apparent reduction in WT(j) from 60 minutes to 15 minutes at station j. However, there have been corresponding increases in WT(i) and WT(k), and the sum of WT changes over all three stations is zero. This assumes that these changes have not changed the sequences at i or k in such a way as to enable a new minimal sequence to be found reducing NAC(i) or NAC(k).

There is an important observation to be made at this point. The quantity WT can only change in discrete increments of 24 hours, and corresponds to a NF reduction of 1 vehicle. This makes the problem non-linear, and explains its intractability to linear optimization methods. This

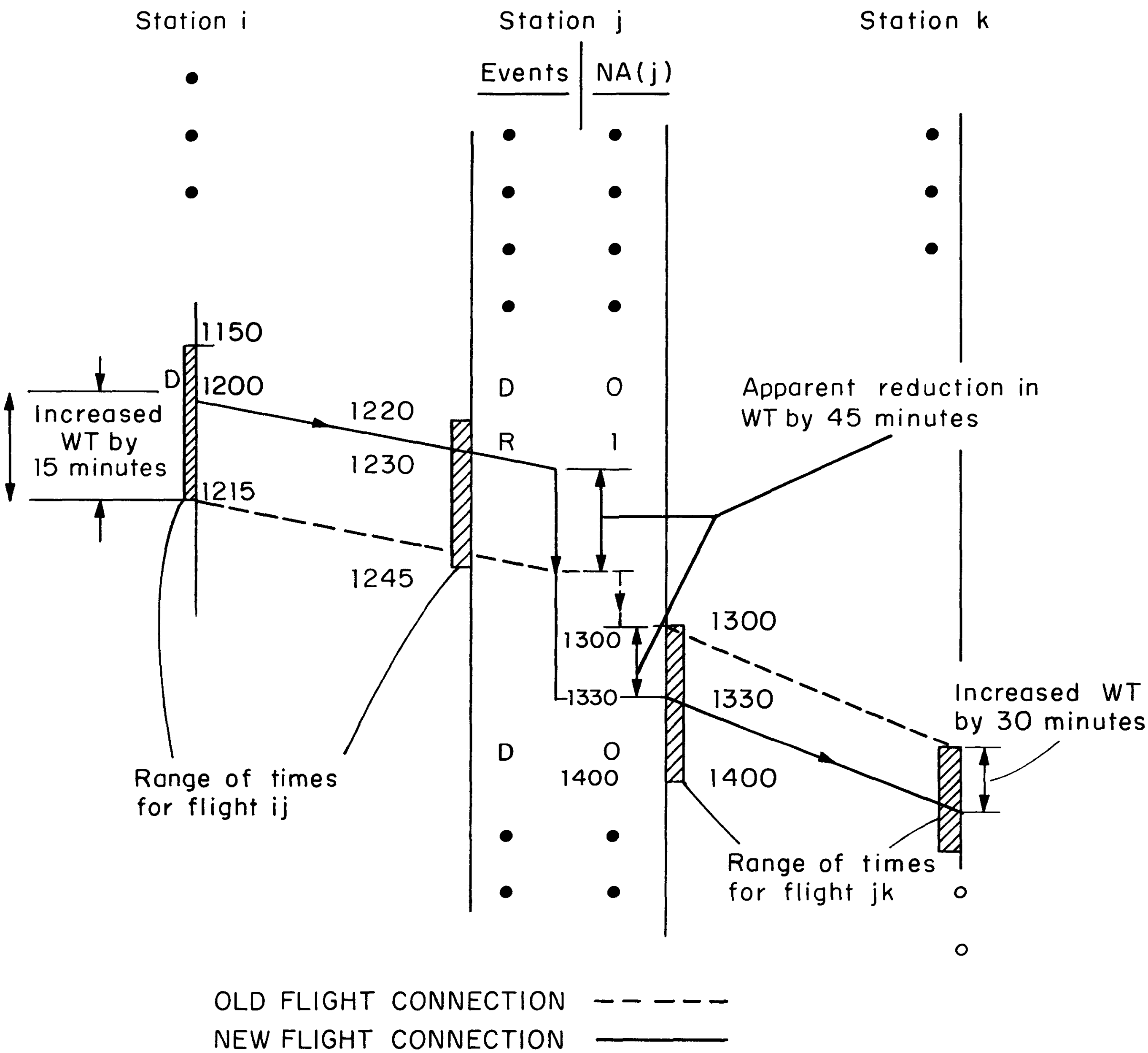


FIGURE 10 EXAMPLE OF APPARENT GROUND TIME REDUCTION

fact can be shown using the previous relations between fleet time, block time, and ground time.

i.e. $24.(NF) = BT + ST + WT$

where ST = total minimal stopping times necessary for load-unload for all services

BT = block time total for a given set of services

Both of these quantities are constant for a fixed schedule.

$\therefore \quad 24.NF = K + WT$

From this relationship, we see that if NF remains constant, WT must be fixed. But NF must be an integer number, and can be reduced in steps of unity. Each unit step will reduce total fleet time by 24 hours, and since BT + ST = K a constant, the reduction must come in WT. Therefore, total WT for any schedule can only be reduced in increments of 24 hours, and corresponds directly to the elimination of one aircraft from the required fleet. It is possible therefore, to concentrate on the elimination of aircraft in order to optimize fleet utilization.

One further observation is that any change made to reduce NAC(j) will connect two flight segments into a continuing flight. The input of services in this study has been individual services consisting of one flight segment, and the segments are only definitely connected when such a change is made. If the input were to be flights of more than one segment, then the connections could be restricted between the segments, and the turns made only at the flight termination; i.e., a flight ABCD can be treated as a flight AD with appropriate times, and the optimization process is similar.

A Statement of the Fleet Reduction Algorithm

1. Examine a station, j, to locate zeroes in the minimal NA(j) sequence.

2. For each zero located, attempt to increase its value to unity by a) moving a later arrival forward in the sequence.

   b) moving the zero departure later in the sequence.

3. (a) Scan the E sequence after the zero departure to locate the next arrival event. Determine time change required to place this arrival 1 minute ahead of zero departure. Check for move within range of times associated with this service, and that any change of sequence at the origin station i does not create a need for more aircraft at that station. If it checks, make the appropriate changes at station j and i which eliminate the zero. If not, continue scanning to locate the next arrival event, and try again.

   (b) If the second arrival is not successful, turn to method 2b, and attempt to move the zero departure to be 1 minute later than either of the two arrivals. (Note that the scanning has been limited to the zero departure and the next two arrival events. It is possible that the scope of this scan for a feasible change should be extended). Check each departure move to ensure that it is within the departure range, and that it does not create a need for more aircraft at the destination station, k. If it checks, make the changes of sequence which eliminate the zero. If not, leave station j, and start from step 1 with station j + 1.

4. If the zero is eliminated, continue examining the NA(j) sequence until either; a) the end of the NA(j) sequence is reached; b) a zero cannot be eliminated.

5. If 4(a) occurs, unity can be subtracted from every member of NA(j). If NAC(j) is greater than 5, the present algorithm returns to step 1 with station j and repeat 1 through 5. If NAC(j) is less or equal to 5, the next station j + 1 is examined starting from step 1.

6. If 4(b) occurs, the next station j + 1 is examined starting from step 1.

7. When j is the last station to be examined, the process can be terminated, or iterated until NF does not decrease during any complete pass.

TABLE 6. SUMMARY OF VEHICLE UTILIZATION FROM TIMETABLE

	Peaked Schedule		Flat Schedule	
	Initial	Final	Initial	Final
Fleet size, NF	251	164	238	121
Utilization - hrs/yr	1190	1820	1249	2460
Utilization - hrs/day	3.26	5.00	3.42	6.75
No. of Vehicle Trips/day ($\overline{BT}$ = 15 minutes)	13	20	14	27

Results of Application of Algorithm to Timetables

There are two distinct timetables associated with a peaking of services to match demand, and a flat distribution of services when load factors are allowed to vary. Table 6 summarizes the pertinent quantities for both timetables before and after the optimization of the schedule. The improvement is quite marked (utilizations are roughly doubled) because the initial choice of times for services did not take into account the connectivity between flights. It shows the sensitivity of the utilization to such considerations, and indicates that dynamic scheduling where passenger demand alone determines service will have poor vehicle utilization. A similar algorithm applied to a real airline schedule assuming $\pm$ ½ hour departure ranges gave only a 10% improvement, and typically times had to be changed for seven different flights over four stations to get rid of just one airplane. An airline scheduler would have tightened the schedule by making good "turns" except at those places where slack was intentionally introduced for schedule reliability, etc.

The effect of peaking is quite marked, especially when utilization has been optimized. Daily utilization of

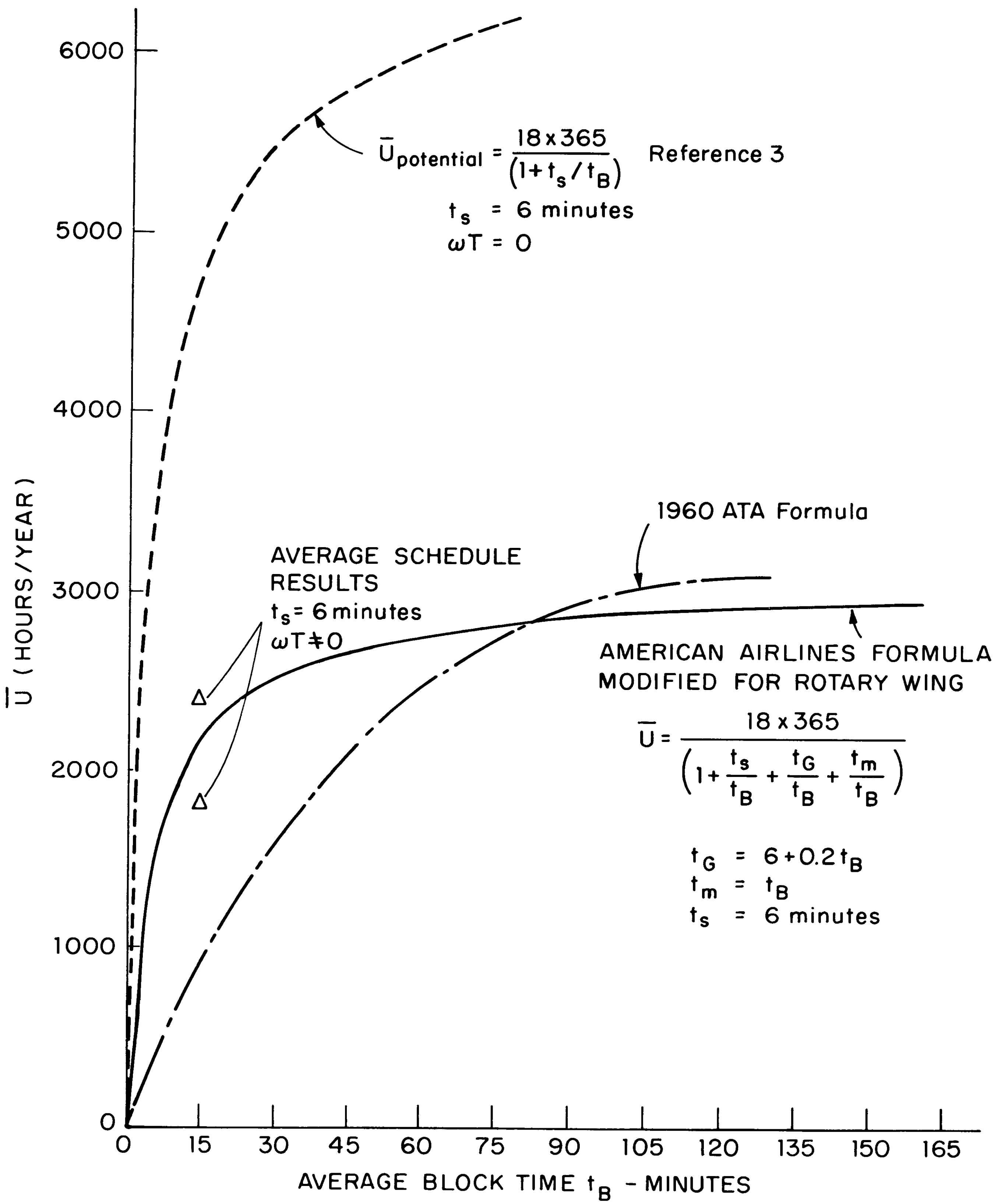


FIGURE 11 VARIATION OF AIRCRAFT UTILIZATION WITH AVERAGE BLOCK TIME

6.75 hours/day drops to 5.00 hours/day for the case where schedules are bunched following the EAL shuttle demand distribution. When the utilization is low, there are apparently sufficient slack airplanes in the schedule to dampen the effect of peak service requirements.

The utilizations achieved are short of the 3000 hours/year value assumed for costing purposes in the initial report, Reference 3. Figure 11 shows some typical estimates for variation of utilization with average block time. The actual results bracket a formula from American Airlines, which corrects the formula used in Reference 3 by adding maintenance check time, and a guard time for variability in operations. The potential utilization would be achieved when a flight could be started immediately after the end of the stopping time. The actual utilization shows the effect of time spent on the ground awaiting a suitable departure time. For example, the average vehicle spends 6.75 hours/day in block time, for the flat demand schedule. It makes about 27 trips/day and has an assumed total stop time for load-unload of about 2.7 hours/day. The remainder of an 18 hour useful airline day, or 8.55 hours/day gives 8.55/27 = 19 minutes as the average time spent waiting for a suitable departure time. Utilization can be increased at the expense of load factor by departing early at less suitable times. The tradeoff is normally not acceptable economically. A detailed econometric model of the airline market and schedule is required to ascertain the marginal revenues and costs involved in such a tradeoff.

There are various ideas which should be investigated to see their effect on achieving higher utilizations:

1) Change nature of peaking throughout the day.
2) Use smaller vehicles, and average passenger loads.
3) Allow variable load factors throughout the day, and in low density markets.
4) Split fleet into two vehicle fleets and use smaller on low density services.
5) Experiment with the optimization algorithm to improve its effectiveness.

VII. RESULTING SCHEDULES AND DISCUSSION

The schedules which have been constructed and optimized are too large to be completely shown in this report. Instead various selected portions are presented to give some indication of the size and detail of these schedules. Figures 12 through 15 give the daily schedule for arrivals and departures for some of the smaller stations in the system; Providence, Philadelphia airport, Hartford, and downtown Boston. Times are given in hundredths of an hour and NA represents the number of aircraft on the ground after each time. The schedule construction that was used for these samples was the peaking distribution where flights are bunched at 9 and 5 o'clock.

The station schedules give arrivals and departures for direct flights or services only. They are useful to give an idea of station loadings to determine personnel and ground facilities requirements, and the peaking in passenger flows. Note that NA is not quite correct in that it describes the time when an incoming arrival could be ready for departure. The actual arrival occurred 0.10 hours (or 6 minutes) earlier, and the number of gates required can be determined using this correction.

However, the listing of arrivals and departures does not give a complete description of the service to any station, in the sense that the flights may be continuing flights. An arrival will represent service from a series of previous cities. To show complete service is a difficult task. As an example, Figure 16 shows the service from the Washington area (two terminals), and includes non-stop and one-stop flights only. There are 86 services per day which is slightly less than the present airline service offered by seven competitors.

Figure 17 shows the schedule for Washington National airport for the peaking and flat schedules, and both before and after the optimization for vehicle utilization. They are shown in graphical form with a vertical time scale so that the peaking effects can be seen, and both the number of changes of flights, and the extent of the time changes can be seen. As can be seen, a relatively few flights are affected, and the time changes involved are of the order of 6 or 7 minutes. The largest change of time is 13.2 minutes.

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Definitions

Route - a path between a city pair followed by vehicles in the system, e.g. A B C P Q. A route segment is a direct link between two cities along a route, e.g. C P in above example. However, C P itself is also a route, or direct route for the cities C and P, so that the terms route and route segment may be used interchangeably.

Service - the offering of transportation between a city pair. A specific service is a flight or combination of flights going between the city pair by any route. The number of such services per day is called the Frequency of Service for the city pair, and the matrix describing frequency of service for all city pairs is called the Frequency Pattern. In this report, the frequency of service and frequency pattern shall include only non-stop services. Other services can be constructed by combining non-stop services.

Flight - a flight is a specific set of services offered by a vehicle following a given route. A _Flight segment_ is a direct hop between cities along the route, and may itself be called a flight or _direct flight_. For example, Flight N at 9:00 am follows route A B C P Q. It provides the following set of services:

Nonstop services A B, B C, C P, P Q

One stop services A C, B P, C Q

Two stop services A P, B Q

Three stop services A Q

Flight segments are A B, B C, C P and P Q. Another flight Flight M may consist of only one flight segment, A Q, i.e., a direct flight A to Q.

Fig. 12 OPERATIONS AT PROVIDENCE

NA	ARRIVALS	TIME	DEPARTURES
2		0734	LaGuardia
1		0751	Mitchell
2	LaGuardia	0752	
1		0753	JFK Airport
2	Mitchell	0754	
1		0755	New Bedford
2	Boston downtown	0793	
1		0794	Boston downtown
2	New London	0804	
1		0805	New London
2	JFK Airport	0842	
1		0865	LaGuardia
2	New Bedford	0906	
1		0907	Mitchell
2	LaGuardia	0916	
3	Mitchell	0955	
2		0958	LaGuardia
1		0973	Hartford
2	LaGuardia	1003	
1		1004	JFK Airport
2	Hartford	1016	
1		1026	Mitchell
2	JFK Airport	1044	
1		1045	LaGuardia
3	Boston downtown, Mitchell	1068	
1		1069	Boston downtown, New London
2	New London	1093	
3	LaGuardia	1096	
2		1136	LaGuardia
1		1148	Mitchell
2	LaGuardia	1187	
3	Mitchell	1196	
2		1239	LaGuardia
1		1252	JFK Airport
2	LaGuardia	1284	
1		1285	Mitchell
2	JFK Airport	1303	
3	Mitchell	1343	
2		1366	LaGuardia
3	Boston downtown	1378	
1		1379	Boston downtown New London
2	New London	1403	
3	LaGuardia	1417	
2		1511	Mitchell
1		1551	LaGuardia
2	Mitchell	1559	
3	LaGuardia	1602	
2		1674	New Bedford
1		1694	JFK Airport
2	New Bedford	1713	
3	JFK Airport	1745	
2		1776	LaGuardia
1		1793	Mitchell
2	LaGuardia	1821	
1		1822	New London
2	Mitchell	1833	
1		1834	Boston downtown
2	Boston downtown	1848	
1		1849	LaGuardia
2	New London	1871	
1		1875	Mitchell
2	LaGuardia	1900	
1		1901	JFK Airport
0		1906	LaGuardia
1	Mitchell	1938	
0		1946	Mitchell
1	JFK Airport	1952	
0		1954	LaGuardia
1	LaGuardia	1958	
0		1970	Boston downtown
1	Mitchell	1994	
0		1995	New London
1	LaGuardia	2001	
0		2002	LaGuardia
2	Boston downtown, New London	2019	
1		2022	Mitchell
0		2025	JFK Airport
1	LaGuardia	2061	
0		2062	Hartford
1	Mitchell	2070	
0		2071	LaGuardia
1	JFK Airport	2076	
2	Hartford	2078	
1		2113	Mitchell
2	LaGuardia	2122	
1		2124	New Bedford
2	New Bedford	2143	
1		2151	LaGuardia
2	Mitchell	2161	
3	LaGuardia	2202	
1		2217	Boston Downtown, New London
0		2234	JFK Airport
2	Boston downtown, New London	2241	
1		2284	Mitchell
2	JFK Airport	2285	
1		2305	LaGuardia
2	Mitchell	2332	
3	LaGuardia	2356	

Fig. 13 OPERATIONS AT PHILADELPHIA AIRPORT

NA	ARRIVALS	TIME	DEPARTURES
5		0714	Philadelphia downtown
5	Philadelphia downtown	0728	LaGuardia
4		0731	Baltimore
3		0738	Washington, DC
2		0741	Mitchell
1		0742	Wilmington
3	Wilmington, LaGuardia	0751	
1		0752	JFK Airport, Newark
2	Baltimore	0766	
1		0770	New York
3	Washington, DC Mitchell	0778	
1		0779	NY Teterboro Airport, Stamford, Greenwich
2	JFK Airport	0801	
1		0805	Lancaster
2	New York	0806	
3	N.Y. Teterboro Airport	0817	
2		0819	Philadelphia downtown
3	Stamford, Greenwich	0823	
4	Newark	0826	
6	Lancaster, Philadelphia downtown	0833	
5		0851	LaGuardia
4		0858	Baltimore
3		0874	Washington, DC
2		0884	Mitchell
1		0888	Philadelphia
2	LaGuardia	0889	
3	Baltimore	0893	
2		0895	Wilmington
3	Philadelphia downtown	0902	
4	Wilmington	0914	
5	Washington, DC	0917	
6	Mitchell	0923	
5		0935	LaGuardia
4		0939	JFK Airport
3		0946	Baltimore
2		0949	Philadelphia downtown
1		0961	New York
2	Philadelphia downtown	0963	
1		0972	Washington, DC
2	LaGuardia	0973	
3	JFK Airport	0977	
3	Baltimore	0981	Stamford, Greenwich
1		0987	Mitchell, N.Y. Teterboro Airport

NA	ARRIVALS	TIME	DEPARTURES
2	New York	0997	
1		1002	Wilmington
2	Washington, DC	1005	
1		1006	Philadelphia
2	Philadelphia downtown	1012	
1		1013	LaGuardia
2	Wilmington	1021	
1		1022	Newark
2	N.Y. Teterboro Airport	1025	
3	Mitchell	1026	
2		1028	Baltimore
3	Stamford Greenwich	1031	
4	LaGuardia	1051	
5	Newark	1057	
4		1062	Lancaster
5	Baltimore	1063	
4		1064	Philadelphia downtown
3		1065	Washington, DC
2		1072	Mitchell
3	Philadelphia downtown	1078	
2		1083	JFK Airport
1		1091	LaGuardia
1		1102	Wilmington
2	Washington, DC	1108	
1		1112	Baltimore
2	JFK Airport	1121	
1		1122	New York
2	Mitchell	1123	
1		1124	Philadelphia downtown
2	LaGuardia	1129	
3	Wilmington	1134	
4	Philadelphia downtown	1138	
5	Baltimore	1147	
6	New York	1158	
5		1164	Washington, DC
2		1175	LaGuardia, NY Teterboro Airport, Stamford, Greenwich
1		1188	Philadelphia downtown
2	Philadelphia downtown	1198	
1		1199	Mitchell
2	Washington, DC	1203	
1		1204	Baltimore
3	LaGuardia, N.Y.	1213	
4	Stamford, Greenwich	1219	
3		1232	Wilmington

Fig. 13 OPERATIONS AT PHILADELPHIA AIRPORT (cont'd)

NA	ARRIVALS	TIME	DEPARTURES
4	Mitchell	1238	
5	Baltimore	1239	
4		1244	JFK Airport
3		1252	Newark
3	Wilmington	1260	Philadelphia downtown
2		1272	LaGuardia
3	Philadelphia downtown	1274	
2		1281	Washington, DC
3	JFK Airport	1282	
4	Newark	1287	
5	LaGuardia	1310	
4		1313	Baltimore
3		1321	New York
4	Washington, DC	1324	
3		1334	Mitchell
2		1344	Philadelphia downtown
3	Baltimore	1348	
4	New York	1357	
5	Philadelphia downtown	1358	
4		1363	Lancaster
5	Mitchell	1373	
4		1392	LaGuardia
3		1406	Wilmington
4	Lancaster	1407	
5	Wilmington	1425	
6	LaGuardia	1430	
5		1435	Washington, DC
3		1441	N.Y. Teterboro Airport, Stamford, Greenwich
2		1449	Philadelphia downtown
1		1458	Baltimore
2	Philadelphia downtown	1463	
3	Washington, DC	1478	
4	NY Teterboro Airport	1479	
3		1482	JFK Airport
4	Stamford, Greenwich	1485	
5	Baltimore	1493	
6	JFK Airport	1520	
5		1533	Mitchell
4		1565	LaGuardia
5	Mitchell	1572	
4		1594	Philadelphia downtown
5	LaGuardia	1603	
6	Philadelphia downtown	1608	
5		1658	Newark

NA	ARRIVALS	TIME	DEPARTURES
4		1674	Wilmington
1		1694	Baltimore, Washington, DC, N.Y.
2	Wilmington	1713	
4	Newark, Baltimore	1729	
5	New York	1730	
6	Washington, DC	1737	
5		1756	Philadelphia downtown
5	Philadelphia downtown	1770	LaGuardia
4		1784	Mitchell
3		1797	Baltimore
2		1805	JFK Airport
3	LaGuardia	1808	
2		1811	NY Teterboro Airport
1		1816	Lancaster
0		1818	Philadelphia downtown
1	Mitchell	1821	
1	Philadelphia	1822	Stamford, Greenwich
2	JFK Airport	1823	
1		1824	Washington, DC
2	Baltimore	1835	
1		1836	Wilmington
2	Wilmington	1840	
1		1841	LaGuardia
2	N.Y. Teterboro Airport	1849	
1		1850	Mitchell
2	Stamford, Greenwich	1860	
1		1861	Philadelphia downtown
2	Washington, DC	1867	
1		1871	New York
2	Lancaster	1873	
1		1874	Baltimore
2	Philadelphia downtown	1875	
3	LaGuardia	1879	
2		1888	Washington, DC
1		1891	LaGuardia
2	Mitchell	1893	
1		1894	Newark
2	New York	1896	
1		1897	Philadelphia downtown
3	Philadelphia downtown, Baltimore	1905	

Fig. 13 OPERATIONS AT PHILADELPHIA AIRPORT (cont'd)

NA	ARRIVALS	TIME	DEPARTURES
1		1906	Wilmington, JFK Airport
2	LaGuardia	1914	
1		1915	Baltimore
0		1923	Philadelphia downtown
2	Wilmington, Washington, DC	1925	
1		1926	Mitchell
2	JFK Airport	1932	
1		1933	NY Teterboro Airport
2	Newark, Philadelphia downtown	1936	Washington, DC
1		1937	LaGuardia
2	Baltimore	1958	
1		1959	Stamford, Greenwich
2	Mitchell	1963	
2	N.Y. Teterboro Airport	1964	Philadelphia downtown
1		1965	New York
2	LaGuardia	1968	
1		1969	Wilmington
2	Stamford, Greenwich	1970	
0		1971	Baltimore, JFK Airport
1	Philadelphia dntn	1976	
1	Washington, DC	1982	Mitchell
1	Wilmington	1983	LaGuardia
2	New York	1986	
1		1987	Washington, DC
0		1991	Philadelphia downtown
1	Baltimore	1998	
0		1999	Lancaster
1	Philadelphia downtown	2013	
1	LaGuardia	2021	Baltimore
2	Lancaster	2023	
2	JFK Airport	2025	Newark
1		2031	LaGuardia
2	Mitchell	2034	
1		2036	Philadelphia downtown
0		2037	Wilmington
1	Washington, DC	2041	
0		2045	Mitchell
2	Wilmington, Philadelphia downtown	2048	
0		2049	N.Y. Teterboro Airport, Stamford, Greenwich
1	Baltimore	2056	

NA	ARRIVALS	TIME	DEPARTURES
1	Newark	2060	Washington, DC
1	LaGuardia	2069	New York
0		2074	Philadelphia downtown
1	NY Teterboro Airport	2079	
0		2080	Baltimore
1	Mitchell	2084	
0		2085	JFK Airport
1	Philadelphia downtown	2088	
0		2089	LaGuardia
1	Stamford, Greenwich	2093	
2	Washington, DC	2103	
3	New York	2105	
4	Baltimore	2115	
5	JFK Airport	2123	
4		2124	Wilmington
5	LaGuardia	2127	
4		2130	Philadelphia downtown
3		2134	Mitchell
3	Wilmington	2143	Washington, DC
4	Philadelphia downtown	2144	
3		2159	Baltimore
2		2166	LaGuardia
3	Mitchell	2173	
4	Washington, DC	2186	
5	Baltimore	2194	
2		2200	Philadelphia downtown
5	LaGuardia	2204	
6	Philadelphia downtown	2214	
5		2217	Lancaster
4		2222	Newark
5	Lancaster	2245	
3		2247	N.Y. Teterboro Airport, Stamford Greenwich
2		2258	New York
1		2268	JFK Airport
2	Newark	2269	
3	N.Y. Teterboro Airport	2285	
2		2290	Wilmington
3	Stamford, Greenwich	2291	
4	New York	2294	
3		2296	Mitchell
2		2301	Washington, DC
3	JFK Airport	2306	
3	Wilmington,	2309	Baltimore
2		2313	LaGuardia
1		2331	Philadelphia downtown
2	Mitchell	2335	
4	Baltimore, Washington, DC	2344	
5	Philadelphia downtown	2345	
6	LaGuardia	2351	

Fig. 14 OPERATIONS AT HARTFORD

NA	ARRIVALS	TIME	DEPARTURES	NA	ARRIVALS	TIME	DEPARTURES
5		0710	New Haven	3		1252	Boston Logan, New York
3		0734	Hartford Bradfield,	4	New Haven	1262	
2		0741	LaGuardia	3		1278	Hartford
1		0749	Newark	4	Boston Logan	1290	
2	New Haven	0757		5	Newark	1293	
3	Hartford, Bradfield	0767		6	Hartford	1311	
2		0770	Philadelphia downtown	4		1321	Philadelphia downtown, Scranton
3	LaGuardia	0779		3		1334	LaGuardia
2		0783	Boston downtown	2		1366	New Haven
1		0791	Boston Logan	3	LaGuardia	1372	
2	Scranton	0826		4	Scranton	1377	
3	Boston Logan	0829		3		1379	Boston downtown
4	Philadelphia downtown	0831		4	Philadelphia downtown	1382	
5	Newark	0832		5	New Haven	1389	
6	Boston downtown	0841		6	Boston downtown	1415	
5		0859	New Haven	5		1494	Hartford, Bradfield
4		0884	LaGuardia	6	Hartford Bradfield	1527	
5	New Haven	0888		5		1533	LaGuardia
4		0890	Hartford	4		1551	New Haven
3		0895	Scranton	5	LaGuardia	1571	
4	LaGuardia	0922		6	New Haven	1574	
5	Hartford	0923		2		1694	Boston Logan, Newark, Philadelphia downtown, Scranton
4		0941	Waterbury	3	Boston Logan	1732	
3		0958	New Haven	4	Newark	1735	
2		0961	Philadelphia downtown	5	Scranton	1750	
3	New Haven	0981		6	Philadelphia downtown	1755	
1		0987	Providence, LaGuardia	4		1776	Hartford, Bradfield, New Haven
2	Providence	1002		3		1784	LaGuardia
1		1003	Boston Logan	4	New Haven	1799	
2	Scranton	1008		5	Hartford, Bradfield	1809	
1		1009	Hartford, Bradfield	6	LaGuardia	1822	
2	Philadelphia downtown	1021		5		1828	Boston downtown
1		1022	Newark	4		1852	New Haven
2	LaGuardia	1025		2		1865	Hartford Bradfield, LaGuardia
1		1032	Boston downtown	1		1868	Philadelphia downtown
2	Hartford Bradfield	1042		2	New Haven	1870	
1		1045	New Haven	1		1871	Scranton
2	Boston Logan	1060		2	Boston downtown	1877	
3	Newark	1063		1		1878	Boston Logan
4	New Haven	1068		2	LaGuardia	1894	
3		1088	LaGuardia	1		1895	Newark
4	Boston downtown	1105		2	Hartford Bradfield	1898	
2		1122	Philadelphia downtown, Scranton	1		1907	New Haven
3	LaGuardia	1126		2	Scranton	1924	
2		1131	Hartford Brad-	1		1925	LaGuardia
3	Waterbury	1135		2	New Haven	1930	
2		1136	New Haven	3	Philadelphia downtown	1932	
3	New Haven	1159		2		1934	Hartford Bradfield
4	Hartford Bradfield	1164		3	Newark	1942	
5	Scranton	1178		2		1945	Waterbury
6	Philadelphia downtown	1183		3	Boston Logan	1955	
5		1199	LaGuardia	2		1958	New Haven
6	LaGuardia	1237		3	LaGuardia	1964	
5		1239	New Haven	1		1965	Philadelphia downtown, Scranton

Fig. 14 OPERATIONS AT HARTFORD (cont'd)

NA	ARRIVALS	TIME	DEPARTURES
2	Hartford Bradfield	1967	
3	New Haven	1981	
2		1984	LaGuardia
1		1995	Boston downtown
2	Scranton	1996	
1		1997	Boston Logan
2	LaGuardia	2009	
1		2010	New Haven
0		2022	Hartford Bradfield
1	Philadelphia downtown	2024	
0		2025	Newark
1	Boston downtown	2031	
2	New Haven	2033	
0		2049	Providence, LaGuardia
1	Hartford Bradfield	2055	
2	Newark	2066	
3	Waterbury	2068	
1		2069	Philadelphia downtown, Scranton
0		2071	New Haven
1	Boston Logan	2074	
2	Providence	2091	
3	LaGuardia	2092	
4	New Haven	2094	
3		2096	Hartford Bradfield
4	Scranton	2125	
5	Hartford Bradfield	2129	
6	Philadelphia down-town	2130	
5		2134	LaGuardia
4		2151	New Haven
5	LaGuardia	2172	
6	New Haven	2174	
5		2217	Boston downtown
4		2226	Newark
3		2234	Boston Logan
4	Boston downtown	2253	
2		2258	Philadelphia down-town, Scranton
1		2267	Hartford, Bradfield
2	Boston Logan	2272	
3	Newark	2275	
2		2296	LaGuardia
3	Hartford Bradfield	2300	
2		2305	New Haven
3	Scranton	2314	
4	Philadelphia down-town	2319	
5	New Haven	2328	
6	LaGuardia	2334	

Fig. 15 OPERATIONS AT BOC - BOSTON ON THE CHARLES

NA	ARRIVALS	TIME	DEPARTURES	NA	ARRIVALS	TIME	DEPARTURES
4		0614	LaGuardia	2	Boston Logan	1368	
3		0656	Boston Logan	1		1369	Boston Logan
2		0669	Providence	3	Newark,Mitchell	1378	
1	Boston Logan	0670	Newark,Mitchell Field	1		1379	JFK Airport, Hartford
2	LaGuardia	0675		2	Providence	1403	
1		0691	Philadelphia	3	LaGuardia	1405	
3	Mitchell,Newark	0704		4	Hartford	1415	
1		0805	JFK Airport, Hartford	5	JFK Airport	1440	
2	Providence	0818		4		1449	LaGuardia
2	Hartford	0819	LaGuardia	5	LaGuardia	1510	
1		0820	Worchester	4		1594	LaGuardia
2	Worchester	0847		3		1636	Fitchburg Airport
3	JFK Airport	0866		4	LaGuardia	1655	
4	Philadelphia	0874		3		1670	Worchester
5	LaGuardia	0880		5	Boston Logan, Worchester	1693	
4		0888	LaGuardia	1		1694	Boston Logan, Newark, Mitchell, Philadelphia
3		0922	Boston Logan	2	Fitchburg Air-port	1751	
4	Boston Logan	0936		3	Mitchell	1752	
4	LaGuardia	0949	LaGuardia	2		1756	LaGuardia
3		0957	Newark	3	Newark	1758	
2		0961	Mitchell	4	Philadelphia	1777	
1		1006	LaGuardia	3		1815	LaGuardia
2	LaGuardia	1010		4	LaGuardia	1817	
3	Mitchell	1019		3		1820	JFK Airport
2		1022	Philadelphia	2		1824	Providence
3	Newark	1025		1		1841	Hartford
2		1034	Boston Logan	2	Boston Logan	1850	
1		1044	Providence	1		1851	Boston Logan
2	Boston Logan	1063		2	Providence	1858	
1		1064	LaGuardia	1		1859	LaGuardia
2	LaGuardia	1067		2	LaGuardia	1860	
3	Hartford	1068		1		1861	Newark
1		1069	JFK Airport, Hart-ford	2	Hartford	1864	
0		1088	Worchester	1		1865	Mitchell
1	Providence	1093		0		1889	LaGuardia
2	Philadelphia	1105		1	JFK Airport	1900	
3	LaGuardia	1121		0		1901	Philadelphia
1		1122	Newark,Mitchell	1	LaGuardia	1914	
2	JFK Airport	1123		0		1915	Boston Logan
1		1124	LaGuardia	1	Mitchell	1929	
2	Worchester	1164		0		1931	LaGuardia
3	Mitchell	1180		1	Newark	1935	
2	Boston Logan	1184		2	Boston Logan	1944	
3	LaGuardia	1185		1		1955	Worchester
4	Newark	1186		2	LaGuardia	1958	
3		1188	LaGuardia	1		1960	Newark
4	Boston Logan	1204		2	Worchester	1962	
5	LaGuardia	1249		1	Philadelphia	1963	JFK Airport, Mitchell
4		1252	Philadelphia	0		1964	LaGuardia
3		1260	LaGuardia	1	LaGuardia	1992	
2	LaGuardia	1321	Newark,Mitchell	2	Providence	1994	
3	Philadelphia	1335		0		1995	Providence, Hartford
2		1344	Logan	2	Logan,Mitchell	2005	
1		1354	Providence	0		2006	Boston Logan, LaGuardia

Fig. 15 OPERATIONS AT BOC - BOSTON ON THE CHARLES (cont'd)

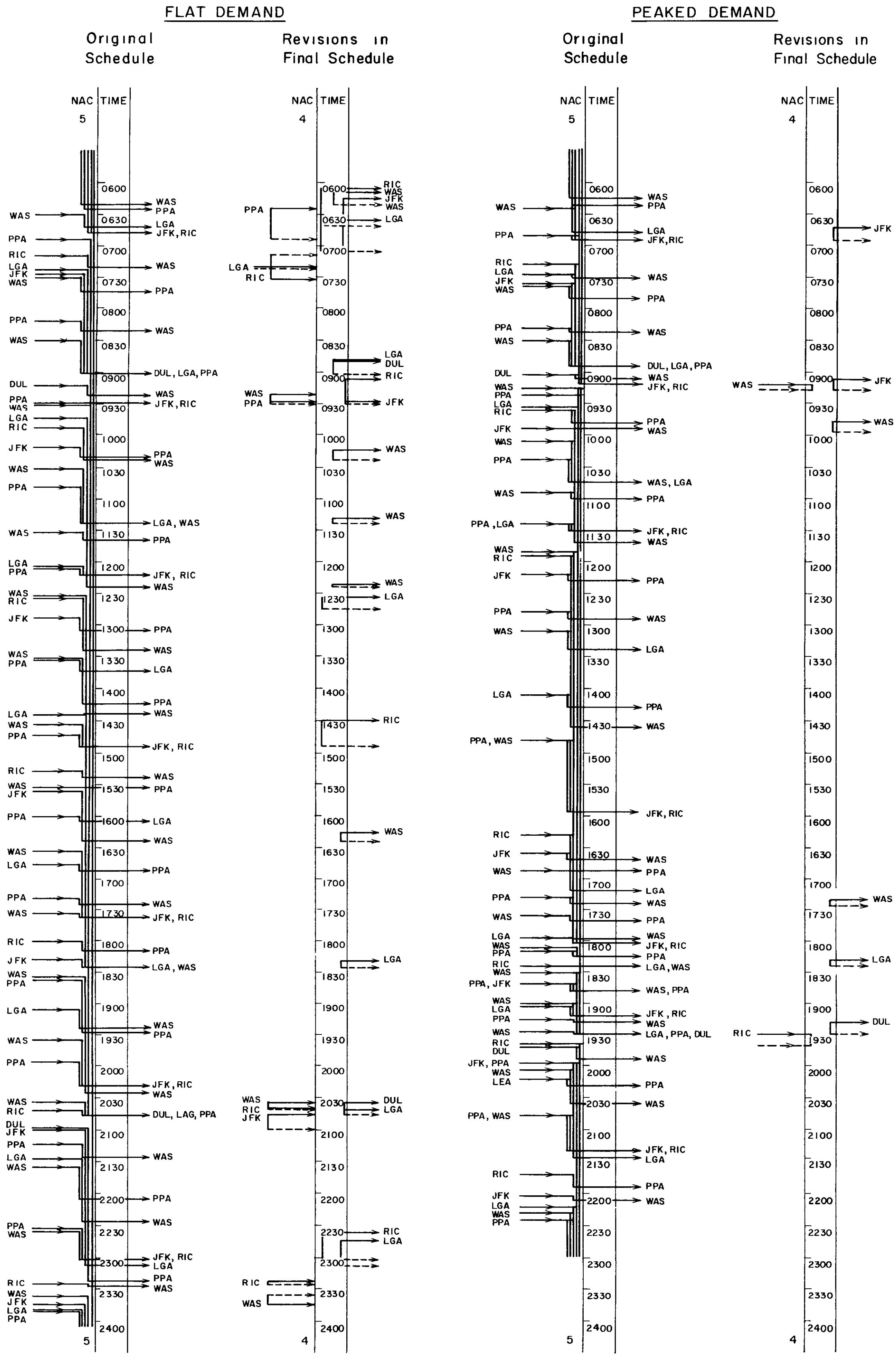
NA	ARRIVALS	TIME	DEPARTURES
1	LaGuardia	2025	Philadelphia
1	Newark	2029	
2	Hartford	2031	
1		2036	LaGuardia
2	JFK Airport	2056	
3	LaGuardia	2060	
1		2069	Newark, Mitchell
0		2078	LaGuardia
1	LaGuardia	2097	
0		2100	Boston Logan
1	Philadelphia	2108	
2	Boston Logan	2114	
3	Mitchell	2127	
2		2130	LaGuardia
3	Newark	2133	
4	LaGuardia	2139	
3		2172	JFK Airport
4	LaGuardia	2191	
3		2193	Worchester
2		2200	LaGuardia
0		2217	Providence, Hartford
1	Worchester	2218	
0		2234	Philadelphia
1	Providence	2241	
2	Hartford	2253	
0		2258	Newark, Mitchell
1	LaGuardia	2261	
0		2277	Boston Logan
1	JFK Airport	2278	
2	Boston Logan	2291	
3	Mitchell	2316	
4	Philadelphia	2317	
5	Newark	2322	
4		2331	LaGuardia
5	LaGuardia	2392	

Fig. 16 WASHINGTON AREA TO NEW YORK AREA SERVICE - PEAK SCHEDULE

Leave Washington			Arrive New York				
WAS	DCA	VIA	JFK	LGA	JRB	EWR	NYC
7-26						7.91	
7-38		PPA		8.37			
	7.41	PPA		8.37			
	7.63			8.33			
	7.69		8.39				
7.79				8.49			
	7.79						8.47
8.05			8.73				
8.74		PPA				9.54	
8.74		PHL		9.73			
	8.84	PPA			9.69		
9.22						9.87	
9.39				10.07			
9.46		BMR				10.44	
9.62		PHL				10.60	
9.87							10.53
	9.87			10.57			
	9.87	PPA	10.68				
	9.92		10.62				
10.28		BMR	11.29				
10.65		PHL					11.58
10.69			11.39				
10.83				11.51			
10.83						11.48	
	10.88	PPA	11.71				
11.12		BMR		12.10			
11.64		PPA		12.52			
	11.75			12.45			
11.75							12.41
12.44				13.12			
12.44						13.08	
	12.52		13.22				
12.81		PPA		13.84			
13.13		BMR		14.01			
	13.34				14.26		
13.79			14.49				
14.35		PHL	15.20				
14.35		PPA					15.17
	14.41			15.11			
14.41							15.07
14.55		BMR		15.37			
14.82				15.50			
14.82						15.47	
16.94		PHL		18.08			
	16.94		17.64				
	17.84	PPA			18.71		
17.96				17.54			
18.00		BMR					18.80
18.05						18.70	
	18.20			18.90			
18.22							18.88
18.24		PPA		19.23			
18.24		PHL					19.07
18.32			19.00				
	18.65	PPA		19.50			
18.70		BMR		19.64			
18.82		PAL		19.75			
18.88		PPA			19.71		
	18.91		19.61				
19.04				19.72			
19.04						19.69	
19.15		BMR				19.94	
	19.18	PPA		20.10			
	19.29			19.99			
19.37							20.03
19.39		PHL		20.21			
19.43		PPA				20.25	
19.66		BMR	20.48				
19.80			20.48				
19.80				20.48			
19.84						20.49	
	19.84	PPA			20.65		
20.19		BMR					21.07
	20.25		20.95				
	20.49			21.19			
20.49							21.15
	20.49	PPA				21.32	
20.85				21.53			
20.85						21.50	
21.59		BMR	22.46				
22.17			22.85				
	22.34		23.04				
	22.47			23.17			
22.47							23.13
22.68				23.36			
22.68						23.33'	

FIG. 17 TYPICAL STATION SCHEDULES – WASHINGTON NATIONAL

112 OPERATIONS PER DAY —— TO AND FROM NEW YORK JFK, NEW YORK LGA, WASHINGTON DOWNTOWN, RICHMOND, PHILADELPHIA DOWNTOWN



APPENDIX A

An Extension of the Network Models to Include Time of Day

A more useful class of network models dealing with transportation problems is defined by extending the geometry of the network into a time dimension. In the geographic model used to describe passenger flow, a node represented a point in geographic dimensions (latitude and longitude, for example). The flow in the network, X, was posed as passengers per day. In this appendix, an extension of this problem is shown where each node now represents a point in both space and time. The passengers are now travelling least time paths along specified individual flights, and making flight connections as appropriate. There are many other uses for such space-time models in flight scheduling. References 6 and 7 describe some applications in the area of flight crew scheduling.

Figure 19 shows a simple model of the Time of Day network model. Stations A, B, C etc. are listed across the page, with a vertical line beneath each station representing the time dimension. Nodes along the station line represent a point in time; e.g. A0600, A0900, etc. These station time nodes are joined by station transit arcs of unlimited capacity whose cost is a time proportional to the time difference of its nodes. The flow in these arcs represents passengers waiting for a suitable departure, and they incur transit ground time costs in waiting.

The service network now consists of every individual flight hop as specified by the schedule construction process. The service arcs are a given flight (e.g. A0900 to C1000) with unlimited capacity, and a time cost equal to flight time plus unloading time. The service network is now a routing diagram commonly used by graphical scheduling methods.

At each flight arrival node, a disembarc goes to the station node (e.g., arrivals at C are connected to C*). The disembarcs have unlimited capacity, and a penalty cost of the order of 24 hours. The flow on the set of disembarcs represents people leaving the station throughout the day.

The demand arcs (e.g. D* A0600, D* A0900) distribute the demand between stations throughout the day. The capacities are set to the estimated demand values, and the demand arcs costs are zero. The daily variations in demand are represented by the discrete inputs of demand at any time interval desired (e.g. hourly). The demand arc creates a need for a flow through the service network along the path

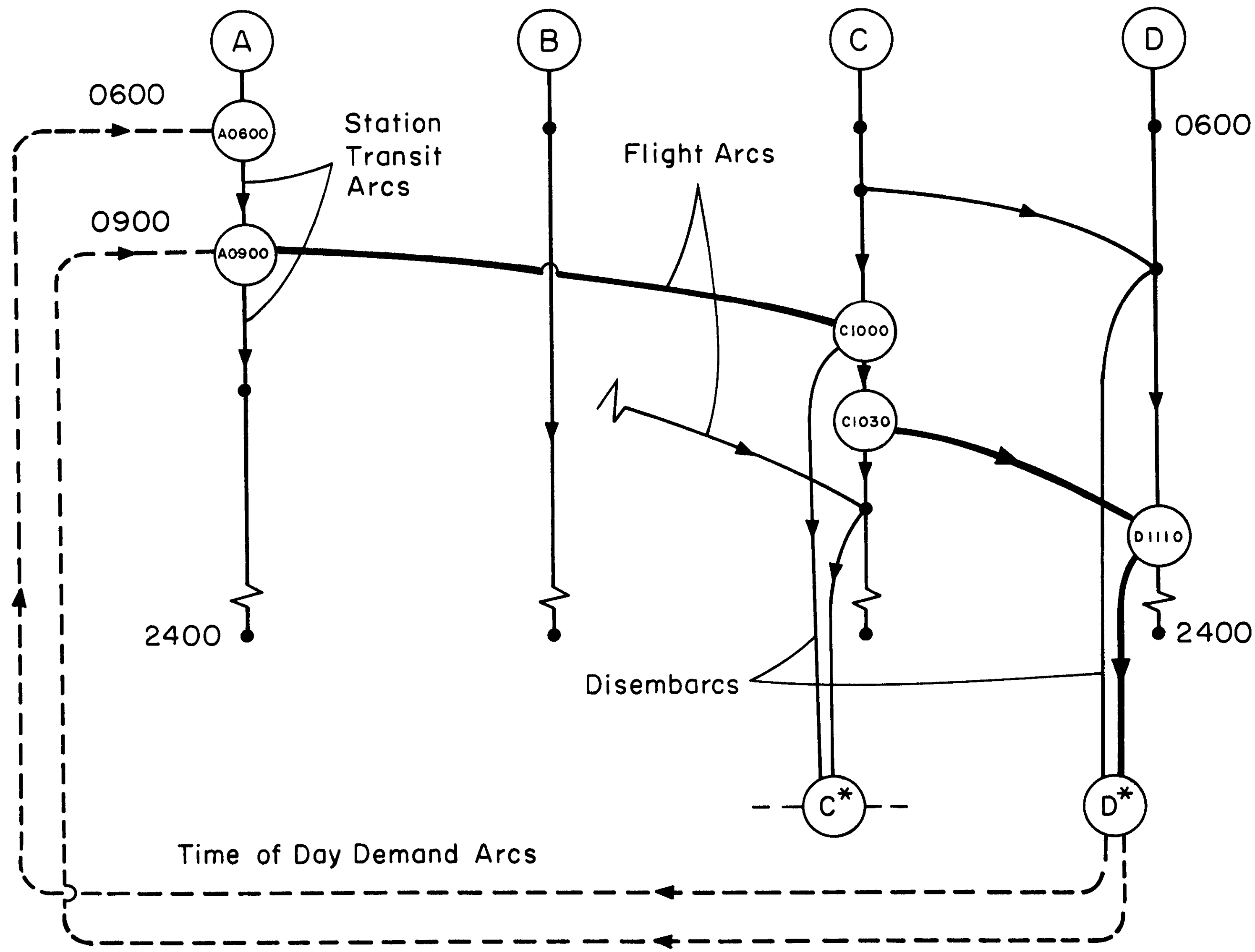


FIGURE 19 A TIME OF DAY TRANSPORTATION NETWORK

which gives least overall trip time.

With the model formulated above, the computer methods used in the geographic model are applicable. The flow solution, X, would describe the number of passengers on board each flight and the number of people on the ground in each station awaiting a departure. A review of passenger load sizes would indicate desirable vehicle size and allow computation of load factor distributions. Any individual flight can be changed in time, and the effect on passenger load noted.

This model has not been exercised on the Northeast Corrider Airbus example. It would be a very large network with some 55000 arcs, and would require reprogramming to handle input of the demand information. It is presented as an example of the use of more detailed network models in schedule planning or schedule control. It can be conceived as a technique to be used in a modal competition problem. For time sensitive passengers, a model having the service networks for each mode, suitably interconnected by transfer arcs for one mode to another at a given city can be constructed, and the passenger flows on each mode obtained. Similar flow solutions can be obtained for cost sensitive passengers, etc. Each mode can then revise its service network to operate more efficiently, or to capture passengers away from other modes. Such a model can also be applicable to study the competition between carriers in a given mode of transportation.

No capacity constraints have been applied to flight service arcs. Such a flow problem is known as the capacitated multi-commodity flow problem for which suitable integer methods of solution do not exist as yet.